Plasma-Immersion Ion Implantation of the Interior Surface of a Small Cylindrical Bore Using an Auxiliary Electrode for Finite Rise-Time Voltage Pulses

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Abstract—Plasma-immersion ion implantation (PIII) can be used to process the interior surfaces of odd-shape specimens such as a cylindrical bore. The temporal evolution of the plasma sheath in a small cylindrical bore in the presence of a grounded coaxial auxiliary electrode is derived for voltage pulses of different rise times by solving Poisson’s equation and the equations of ion continuity, and motion numerically using the appropriate boundary conditions. It is found that the maximum ion impact energy and the average impact energy are improved for finite rise-time voltage pulses, and shorter rise times yield better results. Our results allow the selection of a suitable auxiliary electrode radius to improve the average impact energy for a given rise time.

Index Terms—Ion implantation, plasma applications, plasma sheath.

I. NOMENCLATURE

- $D$: Ion–matrix overlap radius.
- $n_0$: Uniform ion density.
- $n_i$: Ion density.
- $\phi$: Unknown potential.
- $\phi_b$: Target bias.
- $\phi_p$: Peak voltage of the pulse.
- $\varepsilon_0$: Permittivity of free space.
- $c$: Charge of electron.
- $r_b$: Radius of the cylindrical bore.
- $r_a$: Radius of the auxiliary electrode.
- $r$: Radial coordinate.
- $T_e$: Temperature of electron.
- $M$: Ion mass.
- $v_i(0)$: Initial ion velocity.
- $v_i$: Ion velocity.
- $t$: Temporal coordinate.
- $\tau_r$: Rise time of the voltage pulse.
- $k$: Boltzmann constant.
- $\rho$: Normalized radial coordinate.
- $\rho_b$: Normalized radius of the cylindrical bore.
- $\rho_a$: Normalized radius of the auxiliary electrode.
- $\Psi$: Normalized unknown potential.
- $\Psi_b$: Normalized target bias.
- $N_i$: Normalized ion density.
- $V_i$: Normalized ion velocity.
- $T$: Normalized temporal coordinate.
- $T_{\tau_r}$: Normalized rise time of the voltage pulse.
- $v_{\text{max}}$: Maximum ion velocity.
- $\omega_{ps}$: Ion plasma frequency.
- $h$: Grid size in space.
- $f$: Time step.
- $i$: Discrete space coordinate.
- $j$: Discrete temporal coordinate.
- $c$: Constant.
- $E_{i \tau}$: Normalized ion impact energy.
- $E_{i \text{max}}$: Maximum normalized ion impact energy.
- $E_{\text{avg}}$: Average normalized ion impact energy.
- $f(E_i)$: Ion impact energy-distribution function.

II. INTRODUCTION

PLASMA-IMMERSSION ion implantation (PIII) is an innovative and fledgling technique to enhance the surface properties of materials such as metals, polymers, ceramics, and semiconductors [1]–[3]. An appealing feature of PIII is that it is possible to implant surfaces that are not line-of-sight accessible. Interior surfaces of industrial components such as dies, bushings, pipes, and piston rings pose a formidable challenge to conventional ion-beam implantation techniques, and so the problems of inner surface modification using PIII have attracted the attention of physicists and materials scientists [4]–[11].

The structure of the ion–matrix sheath in a transitionally invariant cylindrical bore has been investigated [4], and the crucial parameter is the ion–matrix overlap radius $D$ which is...
given by the following relationship:

\[ D = \sqrt{\frac{-4\pi n_0 \phi_b}{en_0}} \]  

(1)

where \( n_0 \) is the uniform ion density and \( \phi_b \) is the target bias. For zero rise-time voltage pulses and when the radius of a cylindrical bore \( r_b \) is less than or equal to \( D \), the bore is filled by an unneutralized ion component, and can be categorized as a “small bore case.” When \( r_b \) is larger than \( D \), there is a region of neutral plasma around the axis and it is referred to as a “large bore case.” Previous studies have concluded that for PIII in a cylindrical bore, improving the impact energy is most crucial, especially for a small bore \([5, 6]\).

We have proposed to increase the impact energy of ions implanted into the interior sidewalls of cylindrical specimens by using an auxiliary electrode \([11, 12]\). The ion–matrix sheath and temporal evolution of the ion–matrix sheath in a small cylindrical bore have been calculated for zero rise-time voltage pulses, and the impact energy increases significantly in the presence of an auxiliary electrode. However, even though our previous results provide the theoretical foundation, zero rise-time voltage pulses are unrealistic and further investigation must be carried out to incorporate finite rise-time voltage pulses to simulate real experimental conditions. In this paper, our work is focused on the effects of finite rise-time voltage pulses. In this scenario, it can no longer be assumed that there are only ions in the bore and the electron distribution, and the expansion of the plasma sheath in the bore must be considered together.

III. FLUID MODELING AND FORMALISM

We consider PIII into the interior sidewall of a cylindrical bore with an auxiliary electrode, as shown schematically in Fig. 1. The radius of the bore is \( r_b (r_b \leq D) \) and the radius of the auxiliary electrode is \( r_a (r_a < r_b) \). The variable \( r \) measures the distance from the symmetry axis. The system is transitionally invariant along the axis of the bore, and it is azimuthally symmetrical, so that the variables only depend on \( r \). When \( (r_b - r_a) \) is more than the Debye length, we assume that the bore is essentially uniformly filled with a neutral plasma in which the density of electrons and ions are both \( n_0 \) and the potential is zero. The electron is characterized by the electron temperature \( T_e \). The ion mass is \( M \) and the ion charge is \( e \). The initial ion velocity \( v_i(0) \) is zero and the potential in the bore is given by \([13, 14]\).

\[
\phi_r = \begin{cases} \phi_p(t/t_r), & t \leq t_r, \\ \phi_p, & t > t_r, \end{cases}
\]

(2)

It is important to point out that the effects of the fall time of the voltage pulse is negligible as a pulse of sufficient duration will deplete almost all the ions within the bore and there are practically no ions left to experience the fall \([6]\).

This situation is modeled using cold collisionless fluid ions, Boltzmann electrons, and Poisson’s equation \([15]\). In cylindrical coordinates, the equations of ion continuity and motion, Boltzmann’s relationship, and Poisson’s equation are

\[
\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial r} (n_i v_i) = 0
\]

(3)

\[
\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial r} = -\frac{e}{M} \frac{\partial \phi}{\partial r} - \frac{1}{r} \frac{\partial}{\partial r} (\frac{n_i - n_e}{e_0})
\]

(4)

\[
\frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial \phi}{\partial r}) = \frac{e}{e_0} (n_i - n_e)
\]

(6)

where \( 0 < r_a \leq r \leq r_b \leq D \). The variables can be made dimensionless by normalization. The dimensionless variables used in this work are

\[
\rho = \frac{r}{D}, \quad \Psi = \frac{\phi}{\phi_p}, \quad N_i = \frac{n_i}{n_0}, \quad V_i = \frac{v_i}{v_{\max}}, \quad T = t \omega_p
\]

(7)

where \( v_{\max} = \sqrt{-2e\phi_p/M} \) is the velocity the ion would have if it fell through a potential drop \( \phi_p \), and \( \omega_p = \sqrt{k_B T_e/e_0 M} \) is the ion plasma frequency. In this way, the radius of the bore and the auxiliary electrode are \( \rho_b = r_b/D \) and \( \rho_a = r_a/D \), respectively. After substituting (5) into (6), the equations become

\[
\frac{\partial N_i}{\partial T} + \frac{1}{\sqrt{2}} \frac{\partial}{\partial \rho} (\rho N_i V_i) = 0
\]

(8)

\[
\frac{\partial V_i}{\partial T} + \frac{1}{\sqrt{2}} V_i \frac{\partial V_i}{\partial \rho} = \frac{1}{2\sqrt{2}} \frac{\partial \Psi}{\partial \rho}
\]

(9)

\[
\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho \frac{\partial \Psi}{\partial \rho}) = 4(N_i - \text{exp}(\phi_p \Psi/KT_e))
\]

(10)

where \( 0 < \rho_a \leq \rho \leq \rho_b \leq 1 \). Equation (2) also becomes

\[
\Psi_t = \begin{cases} \frac{T}{T_r}, & T \leq T_r, \\ 1, & T > T_r \end{cases}
\]

(11)

where \( T_r = t_r \omega_p \) is the dimensionless rise time.
Equations (8)–(10) are solved by finite-difference methods. The discrete finite-difference equations in the presence of an auxiliary electrode are

\begin{align}
N_{i,j+1} &= N_{i,j} - \frac{f}{\sqrt{2}\hbar\rho + \ii \hbar} \cdot \left[ h N_{i,j} V_{i,j} + (\rho_0 + \ii \hbar)(2N_{i,j} V_{i,j} - N_{i,j} V_{i+1,j}) \right] \\
V_{i,j+1} &= V_{i,j} + \frac{f}{2\hbar} \left[ \Psi_i - \Psi_{i+1} \right] - 2(V_{i,j})^2 + 2V_{i,j} V_{i+1,j} \\
\Psi_{i+1}(\rho_0 + \ii \hbar) + \Psi_i(h - 2\ii \hbar - 2\rho_0) + \Psi_{i-1}(\rho_0 + \ii \hbar - h) &= 4\hbar^2(\rho_0 + \ii \hbar)(N_{i,j} - \exp(-c\Psi_i)) \tag{14}
\end{align}

where \( c = (e\phi|/KT_\sigma) \), \( \hbar \) is the grid size in space, i.e., \( \rho_{j+1} - \rho_j = \hbar \), \( f \) is the time step used in our iteration, i.e., \( T_{j+1} - T_j = f \), and \( i \) and \( j \) are positive integers. \( N_{i,j}, V_{i,j}, \) and \( \Psi_i \) are the normalized ion density, ion velocity, potential, and normalized radii \( \rho = \rho_0 + \ii \hbar \), and time \( T = jf \), respectively. Note that the normalized potential \( \Psi \) is not a direct function of \( T \) except at the target, and only \( N \) and \( V \) are partially differentiated with respect to \( T \), as shown in (9) and (10). However, \( \Psi \) will indeed change with time as \( N \) will vary with \( T \) as indicated in (10) and (14). The boundary conditions are: \( \Psi_0 = 0 \) and \( N_0 = 0 \) at \( \rho = \rho_0 \), and \( \Psi_{i-1}(\rho_0 - \rho_0)/h = \Psi_0 \) at \( \rho = \rho_0 \). The initial conditions are: \( \Psi_i = 0 \), \( V_{i,0} = 0 \), \( N_{i,0} = 1 \) for \( i > 0 \).

Equation (14) is solved via numerical iteration by first letting \( \phi \) be an initial solution and linearizing (14) about this new \( \psi \), and iterate until the process converges. Typically, it takes two–three iterations to converge.

The solved potential \( \Psi_i \) is then used to determine the normalized ion velocity \( V_{i,j+1} \) and ion density \( N_{i,j+1} \) of the next time step \( j+1 \) using (12) and (13). The process of solving the potential at a time step as well as determining the ion velocity and ion density at the next time step is continued until the end of the simulation. A mesh of 200 grids in space and a time step \( f = 0.001 \) is used in my simulation. Therefore, \( fT \) is equal to (1.0–0.1)/200 or 0.0045 when an auxiliary electrode is present \( (\rho_0 = 0.1) \). For \( |\Phi_0| = 30 \text{ kV} \) and \( kT_\sigma = 5 \text{ eV} \), \( c \) is equal to \( 10^4 \).

\[
\Psi_{i+1}(\rho_0 + \ii \hbar) + \Psi_i(h - 2\ii \hbar - 2\rho_0 - 4\hbar^2(\rho_0 + \ii \hbar)\exp(-c\phi) + \Psi_{i-1}(\rho_0 + \ii \hbar - h) = 4\hbar^2(\rho_0 + \ii \hbar)(N_{i,j} - \exp(-c\phi) - c\phi\exp(-c\phi)), \tag{16}
\]

IV. RESULTS AND DISCUSSION

To study the finite rise-time effects, the radii of the bore and the auxiliary electrode are first fixed at \( \rho_0 = 1 \) and \( \rho_0 = 0.1 \), and the rise time is varied \( (T_r = 0, 1, 3, 5, 10, 20, \text{ and } 40) \). The results by the fluid model for \( T_r = 0 \) (zero rise time) are first compared to our previously reported solutions [12] and agree to better than 0.1\%. The fluid model results in the case without the auxiliary electrode are generated for \( T_r = 20 \) and 40 and compared to the results in the literature [6]. The agreement is found to be better than 0.2\%. In order to further corroborate our model and algorithm, simulation is also carried out by the particle-in-cell (PIC) method. In the PIC study, the grid spacing is 0.25 Debye length, the time step is 1/256 \( \omega_{\text{pi}}^{-1} \) and 64 ion particles are used in each cell initially. The results generated by these two distinctly different methods are consistent, thereby confirming the validity of the fluid model.

The relationship between the dimensionless ion density at the target \( N_{\text{at}} \) and the rise time is displayed in Fig. 2. In all cases, there is a steep initial decay in the target ion density, but the rate of decrease slows down as the sheath propagates across the bore. Finally, the faster ions catch up to the slower ones to give rise to the peaks, and \( N_{\text{at}} \) then decreases precipitously to zero. It is obvious that the time needed to deplete all ions increases with \( T_r \). It can also be observed that for a large rise time, e.g., \( T_r \geq 10 \), the ions are exhausted when the potential at the target reaches the plateau value. Based on our results, the peak attains the maximum value when \( T_r = 3 \). It indicates that the ions close to the auxiliary electrode are accelerated
Fig. 3. Normalized ion impact energy \( E_{i,n} \) versus normalized time \( T \) for normalized rise times \( T_r = 0, 1, 3, 5, 10, 20, \) and 40. The dimensionless variables are described in (7), \( \rho_a = 0.1 \) and \( \rho_b = 1.0 \).

Fig. 4. Rise-time dependence of the normalized maximum ion impact energy \( E_{i,\text{max}} \) and the normalized average ion impact energy \( E_{i,\text{avg}} \). The dimensionless variables are described in (7), \( \rho_a = 0.1 \) and \( \rho_b = 1.0 \).

better, and thus more ions are able to catch up to the slower ones. This point can be verified by observing the normalized ion impact energy \( E_{i,t} \) for the different rise times, as exhibited in Fig. 3. The normalized ion impact energy is given by

\[
E_{i,t} = \frac{1}{2} M \frac{\dot{v}_t^2}{\dot{\phi}_p} = \frac{1}{2} M \frac{u_{i,t}^2}{\dot{\phi}_p} = V_{i,t}^2. \tag{17}
\]

Here, \( \dot{v}_t \) is the ion velocity at the target. As the ions are initially at rest, the impact energy starts at zero, increases monotonically with time, and finally reaches the maximum. This is very different from the case without the auxiliary electrode, in which case the impact energy decays after the sheath reaches the axis [6]. It is interesting to note that the maximum ion impact energy is not achieved at \( T_r = 0 \), but rather at \( T_r = 3 \). This point is further illustrated in Fig. 4, which depicts the maximum normalized ion impact energy \( E_{i,\text{max}} \) and the average normalized ion impact energy \( E_{i,\text{avg}} \), i.e., implant energy per ion for different \( T_r \). For \( \rho_b = 1 \) and \( \rho_a = 0.1 \), it is found that the ions near the auxiliary electrode will get the maximum ion impact energy because they experience a longer accelerating time and a bigger average accelerating electric field. The effects of the rise time \( T_r \) on the maximum ion impact energy can be visualized in two ways. The electric field between the bore and the auxiliary electrode increases rapidly for a short \( T_r \) and, consequently, the maximum ion impact energy is improved. On the other hand, the electric field is bunched near the target, and because many ions are still in the bore for a short \( T_r \), the benefits to the maximum ion impact energy are not as substantial. Hence, the ion impact energy does not peak at \( T_r = 0 \), but at \( T_r = 3 \). As shown in Fig. 4, the difference of the maximum ion impact energy is small for \( T_r = 0-5 \).

The average impact energy goes up with decreasing rise time because the faster rise time accelerates the majority of the ions better. It can be observed that the decay rate of the average impact energy is larger in the range \( 0 < T_r < 5 \) than when \( T_r > 5 \) and becomes very mild at \( T_r > 20 \). We believe that there are still some ions which will acquire better acceleration in the bore when the voltage pulse reaches the plateau for \( 0 < T_r < 5 \). The shorter \( T_r \) implies that more ions get better acceleration and so the decay rate of the average impact energy is steep. When the rise time is large, the ions have already been depleted from the bore as the voltage pulse reaches the plateau. A larger \( T_r \) matches a larger ion depletion time (Fig. 2) and it implies that some ions can still be accelerated even though the voltage pulse rises slowly. Thus, the average impact energy is not sensitive to the rise time when \( T_r > 10 \).

In order to understand the ion energy distribution, the time-integrated ion impact energy distribution function \( f(E_{i,t}) \) for different values of \( T_r \) are plotted in Fig. 5, where the distribution is normalized such that the integral over the impact energy gives the total normalized dose. The population of energetic ions is larger for a shorter \( T_r \). This is because the accelerating electric field increases very rapidly for a small \( T_r \) and more ions experience better acceleration.

In order to study the rise-time effects for different auxiliary electrode radii, we have performed the simulation for \( \rho_b = 0 \) (without an auxiliary electrode), 0.2, and 0.5. Fig. 6 plots the normalized average ion impact energy \( E_{i,\text{avg}} \) versus rise time \( T_r \) for \( \rho_a = 0, 0.2, \) and 0.5. It can be observed that when the rise time is short, the average ion impact energy can be enhanced by using a larger auxiliary electrode, but not so for a longer rise time. When \( T_r \) and \( \rho_a \) are large, the average ion impact energy is in fact smaller than that when there is no auxiliary electrode (\( \rho_b = 0 \)). When \( T_r \) is small, there are still a copious number of ions within the bore when the bias has plateaued, and the average ion impact energy depends primarily on the strength and uniformity of the electric field between the bore and the auxiliary electrode. Because the electric field is larger and more uniform for a larger \( \rho_a \), the average impact energy is naturally higher for a larger auxiliary electrode radius. On the contrary, when \( T_r \) is long,
most of the ions have already depleted from the bore before the voltage pulse plateaus, and the average impact energy is mainly dependent on the average potential before the ions are depleted. The smaller the auxiliary electrode radius, the larger is the spacing between the auxiliary electrode and the interior surface of the cylindrical bore sample, and the longer time the ions will have to experience the accelerating electric field. In addition, the accelerating voltage is continuously increasing until the plateau and ions which remain in the bore longer (i.e., smaller $r_B$) will attain a higher net-impact energy. Hence, the average impact energy is larger for a smaller auxiliary electrode radius when $T_r$ is long. Consequently, the choice of the electrode radius depends on the rise time of the pulsing electronics and, in general, a smaller $\rho_a$ is preferred for a reasonable rise time.

V. CONCLUSION

The maximum ion impact energy and the average energy are enhanced by using a grounded auxiliary electrode positioned along the central axis of a cylindrical bore sample for the more realistic finite rise-time voltage pulses. For example, consider a maximum target bias of $-100$ kV applied to a cylindrical bore with a radius equal to the ion–matrix overlap length ($r_B = 1$). If an auxiliary electrode is used ($\rho_a = 0.1$) and $T_r = 5$, the maximum ion impact energy is $82$ kV and the average impact energy is $26$ kV. However, in the absence of the auxiliary electrode and $T_r = 5$, the values will be $33$ and $19$ kV, respectively. When the rise time is lengthened to $T_r = 40$, the effects of the auxiliary electrode are not as pronounced. For instance, the maximum ion impact energy changes from $34$ to $22$ kV and the average impact energy drops from $11.3$ to $11$ kV. Our results thus suggest that a faster voltage rise time is essential to attain good ion impact energy. When a high slew-rate pulsing power supply is not available, the choice of the auxiliary electrode becomes important, as a smaller radius does not necessarily give rise to high impact energy. Our model provides a means to calculate the best auxiliary electrode radius for a given rise time in order to achieve the maximum average ion impact energy.

REFERENCES

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