
Normal Form Equations for Phase Transitions in a Thin Strip

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We consider the deformation of a thin strip composed of a hyperelastic material. To ensure phase transitions can take place, we suppose that the strain energy function is non-convex so that there is a local maximum and a local minimum in the uniaxial stress-strain curve under a homogenous constant strain state. Starting from the three-dimensional field equations, through a novel approach which combines series expansions and asymptotic expansions, we derived two asymptotic model equations which takes into account the lateral deformation and satisfies the traction-free boundary conditions up to the right order for phase transition problem in a thin strip. Since these two equations are derived in a mathematically consistent manner and contain all the required terms to yield the leading-term behavior of the original three-dimensional problem, we call them to be the normal form equations of the governing three-dimensional nonlinear PDE's. We have also given a derivation of the classical Euler's equation through a mathematically consistent approach. Then, we present some interesting analytic solutions for an infinite long strip under free-end boundary condition, including those which seem to describe the structure of a phase boundary, especially the skew characteristic in the lateral direction. By considering the eigenvalue equation for the lateral deformation, it is revealed that the transforming process may accompany both symmetrical and anti-symmetrical lateral deformation and the incline direction of interface of two phase boundaries will change between coincide and opposite alternately. These key features are observed in some experiments.