
Post-Bifurcation Analysis of a Thin-Walled Hyperelastic Tube under Inflation

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Inflation of membrane tubes and the associated problem of bifurcation and instability is a classical subject that has been studied in many books and papers. This problem shares the same features as a family of other problems such as propagating buckles in long metal tubes under external pressure, propagating necks in some polymeric materials when pulled in tension, phase transition, and kink band formation and propagation in layered structures and fibre-reinforced composites. Whereas the determination of the propagation pressure through the equal-area rule is now universally accepted, there is still some uncertainty concerning the determination of the pressure (the *initiation pressure*) at which bulging first occurs. Kyriakides and Chang (1990) took this pressure to be the limiting pressure corresponding to the first turning point in the pressure-volume diagram for uniform inflation, but this value was found to be slightly higher than what had been observed in the experiments. Existing instability and bifurcation results were discussed by Kyriakides and Chang (1990, 1991) but were found not to be relevant to the determination of the initiation pressure. This is probably because neither the bifurcation nor the stability analysis that has been conducted so far is able to describe correctly the bifurcation mode having zero wave number and it is in fact this mode that is relevant to the determination of the initiation pressure. This special mode is not sinusoidal — its correct variation can only be found from a near-critical nonlinear analysis.

In my talk, I shall present a simple procedure that can be used to derive the bifurcation condition and to determine the near-critical behaviour analytically. It is shown that the axial variation of near-critical bifurcated configurations associated with the bifurcation at zero mode number is governed by a first-order differential equation that admits a locally bulging or necking solution. This result suggests that the corresponding bifurcation pressure can be identified with the initiation pressure. This is supported by good agreement with one set of experimental results. It is also shown that the Gent material model can support both bulging and necking solutions whereas the Varga and Ogden material models can only support bulging solutions. Relevance of the present method to the study of nonlinear wave propagation in a fluid-filled distensible tube is also discussed.

References

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