Dynamic Depletion of Vortex Stretching and Nonlinear Stability of 3D Incompressible Flows

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Whether the 3D incompressible Euler or Navier-Stokes equations can develop a finite time singularity from smooth initial data has been a long standing open problem. Here we review some existing computational and theoretical work on possible finite blow-up of the 3D Euler equations. We show that the geometric regularity of vortex filaments, even in an extremely localized region, can lead to dynamic depletion of vortex stretching, thus avoid finite time blowup of the 3D Euler equations. Further, we perform large scale computations of the 3D Euler equations to re-examine the two slightly perturbed anti-parallel vortex tubes which is considered as one of the most attractive candidates for a finite time blowup of the 3D Euler equations. We found that there is tremendous dynamic depletion of vortex stretching and the maximum vorticity does not grow faster than double exponential in time. The stabilizing effect of convection for the 3D incompressible Euler and Navier-Stokes equations will be considered. Finally, we present a new class of solutions for the 3D Euler and Navier-Stokes equations, which exhibit very interesting dynamic growth property. By exploiting the special nonlinear structure of the equations, we prove nonlinear stability and the global regularity of this class of solutions.