Identification of General complex Dynamical Network with Time Delay

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Synchronization of complex networks of coupled dynamical systems has recently received increasing attention and been a hot issue for the last few years. Nowadays, the research on complex dynamical network has been mostly focused on its dynamic behavior and synchronization with knowing its topological structure. However, in many practical situations, the topological structure of complex network is likely to be uncertain, or merely partial known beforehand. Therefore, it is a challenging and forefront issue to propose an effective method for identifying the network topological structure and system parameters together, based on the information of nodes’ dynamics in the network. What's more, because time delays are quite ubiquitous in natural and man-made systems, caused by finite signal transmission speeds or memory effects, one should also consider the influences of time delays in complex network models.

Based on the synchronization theory and adaptive control techniques, an effective approach is then proposed to identify the network topological structure and system parameters together, based on the estimation criteria for determining the global synchronization based on the parameters identification is established.

Consider another complex dynamical network (response network) with coupling delay as follows:

\[ \hat{x}_i(t) = f(t, \hat{x}_i(t)) + F(t, \hat{x}_i(t))\hat{\alpha} + \sum_{j=1}^{N} \hat{c}_{ij} A \hat{x}_j(t - \tau(t)) + u_i, \quad i = 1, \ldots, N \]  \hspace{1cm} (2)

where \( x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in R^n \) is the state vector of the \( i \)-th node, and \( \tau(t) \) is the time-vary delay. \( C = (c_{ij}) \in R^{N \times N} \) is the unknown or uncertain weight configuration matrix. If there is a link from node \( i \) to node \( j \) \( (j \neq i) \), then \( c_{ij} \neq 0 \) and \( c_{ij} \) is the weight; otherwise, \( c_{ij} = 0 \). \( A : R^n \rightarrow R^n \), is inner-coupled matrix.

Consider another complex dynamical network (response network) with coupling delay as follows:

\[ \dot{x}_i(t) = f(t, x_i(t)) + F(t, x_i(t))\alpha + \sum_{j=1}^{N} c_{ij} A x_j(t - \tau(t)) + u_i, \quad i = 1, \ldots, N \]  \hspace{1cm} (1)

where \( x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in R^n \) is the state vector of the \( i \)-th node, \( \tau(t) \) is the time-vary delay. \( C = (c_{ij}) \in R^{N \times N} \) is the unknown or uncertain weight configuration matrix. If there is a link from node \( i \) to node \( j \) \( (j \neq i) \), then \( c_{ij} \neq 0 \) and \( c_{ij} \) is the weight; otherwise, \( c_{ij} = 0 \). \( A : R^n \rightarrow R^n \), is inner-coupled matrix.

Assumption 1(A1). Time-varying delay \( \tau(t) \) is differential function with \( \dot{\tau}(t) \leq \mu < 1 \). Obviously, this assumption is certainly ensured if \( \tau(t) \) is a constant.

Assumption 2(A2). Suppose that there exists a nonnegative constant \( L_1 \) satisfying

\[ ||\dot{f}(t, x(t), \alpha) - \dot{f}(t, y(t), \alpha)|| \leq L_1 ||x(t) - y(t)|| \]

Based on the LaSalle invariance principle of functional differential equations, the following sufficient criteria for determining the global synchronization based on the parameters identification is established.

Theorem 1. Suppose that (A1) and (A2) hold. The weight configuration matrix \( C \) of weighted complex dynamical network with time-vary delay in couplings can be identified by the estimation \( \hat{C} \), and \( \alpha \) can be
identified by the estimation $\hat{\alpha}$, using the following response network:

$$\begin{align*}
\dot{\hat{x}}_i &= f(t, \hat{x}_i(t)) + F(t, \hat{x}_i(t))\hat{\alpha} + \sum_{j=1}^{N} \hat{c}_{ij} A \hat{x}_j(t - \tau) + u_i \\
u_i &= -k_i \hat{x}_i(t), \hat{k}_i = d_i ||\hat{x}_i(t)||^2 \\
\dot{\hat{\alpha}} &= -\sum_{i=1}^{N} F^T(t, \hat{x}_i(t)) \hat{x}_i(t); \hat{c}_{ij} = -\delta_{ij} \hat{x}_i(t)^T A \hat{x}_j(t - \tau)
\end{align*}$$

where $i, j \in V = \{1, ..., N\}$, the constants $d_i$ and adaptive gains $\delta_{ij}$ are any positive constants. They can be chosen properly to adjust the synchronization speed.

For the case of the complex dynamical network (1) coupled by different node dynamics, we also have well considered and got a similar result. Because of the limitation of the length, it is omitted here.

As the complex dynamical network with nodes’ dynamics delay, it can be described as follows:

$$\dot{x}_i(t) = \bar{f}(t, x_i(t), x_i(t - \tau), \alpha) + \sum_{j=1}^{N} c_{ij} A x_j(t), \quad i = 1, ..., N$$

By the similar method, a sufficient criteria for identification of the weight configuration matrix $C$ of complex dynamical network and the unknown parameter vector $\alpha$ of the dynamic system is established. It’s effective for the case of the complex network consisting of nodes with different dynamics as well.

Finally, several typical simulations are used to verify the effectiveness of the proposed approach. It is an online method and can monitor the dynamical evolution of network topological structures and system parameters.