
Non-Uniqueness and Non-Smooth Solutions in the Nonlinear Elasticity of Fibre-Reinforced Solids

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In this talk we study the finite deformation plane strain problem of azimuthal shear of a circular cylindrical tube of incompressible transversely isotropic elastic material. The preferred direction associated with the transverse isotropy lies in the considered plane and is disposed so as to preserve the cylindrical symmetry. For a general form of strain-energy function we obtain a simple expression for the azimuthal shear stress and the associated strong ellipticity condition. For particular choices of strain energy ellipticity does not fail when the preferred direction undergoes extension, but under compression may fail on a (cylindrical) surface of discontinuity of the deformation gradient. The emergence of non-unique solutions, and the existence of non-smooth solutions is examined. It is shown that after a critical value of the applied shear stress there are multiple solutions, but the global energy minimizer corresponds to a non-smooth solution.