Absolute Ruin in the Compound Poisson Risk Model with Constant Dividend Barrier

Haili YUAN, Yijun HU
School of Mathematics and Statistics
Wuhan University, P.R. China
Email: hailiyuan@gmail.com, yijunhu@public.wh.hb.cn

In this paper, we investigate the absolute ruin in the classical compound Poisson risk model with interest and a constant dividend barrier. Absolute ruin means that when the surplus process is below zero, the insurer could borrow money at a debit interest rate to continue her business. Meanwhile, the insurer will repay the debts from her premium income. We assume that an insurer is allowed to continue her business with debts until her debts or negative surplus is below some certain level, and in the latter case, the insurer is no longer allowed to run her business, absolute ruin occurs at this situation.

We assume that the premium rate is \( c (c > 0) \). Let \( \{Y_i; i \geq 1\} \) be i.i.d. positive random variables representing the successive individual claim amounts. Let the total number of claims up to time \( t \), denoted by \( N(t) \) and independent of \( \{Y_i; i \geq 1\} \), be a Poisson process with parameter \( \lambda (\lambda > 0) \). The aggregate claim amount process is defined by \( S(t) := \sum_{i=1}^{N(t)} Y_i; t \geq 0 \) where \( S(t) = 0 \) if \( N(t) = 0 \). \( b (b > 0) \) represents the constant dividend barrier, \( \epsilon (\epsilon \geq 0) \) is the constant force of interest and \( \delta (\delta > \epsilon) \) is the debit interest rate. When the surplus of an insurer \( R_t \) is below zero, the insurer can borrow money at the rate \( \delta \), but when the surplus is below \( -\frac{c}{\delta} \), the insurer can’t repay all her debts from her premium income, in the latter case, the insurer is no longer allowed to run her business. The surplus \( R_t \) is described as

\[
\frac{dR_t}{dt} = \begin{cases} 
-dS_t, & R_t = b, \\
c dt + \epsilon R_t dt - dS_t, & 0 \leq R_t < b, \\
c dt + \delta R_t dt - dS_t, & -\frac{c}{\delta} < R_t < 0.
\end{cases}
\tag{1}
\]

For risk model (1), the time to ruin, denoted by \( T \), is defined as

\[ T := \inf \{t \geq 0; R_t < 0\} \tag{2} \]

and the time to absolute ruin as

\[ T_\delta := \inf \{t \geq 0; R_t < -\frac{c}{\delta}\} \tag{3} \]

First, by a differential argument, an integro-differential equation satisfied by the absolute ruin probability, the distribution and moments of deficit at the time to absolute ruin is derived. In the case of exponential individual claim, the explicit expressions, respectively, are

\[
P(T_\delta < \infty) = 1, \tag{4}
\]

\[
E[I(T_\delta < \infty)|R_0 = x] = e^{-\beta(z - \frac{c}{\delta})}, \quad z > \frac{c}{\delta}, \tag{5}
\]

\[
E[I(T_\delta < \infty)|R_0 = x] = \sum_{k=0}^{n} \frac{n!}{(n-k)!\beta^k} \left( \frac{c}{\delta} \right)^{n-k}, \tag{6}
\]

where \( 1/\beta \) is the mean of the exponential claim.

Finally, the probability of recovery from certain negative surplus level to surplus level zero is also discussed. Denote by \( \hat{T} \) the time of recovery for risk model (1)

\[ \hat{T} := \inf \{t > 0, R(T + t) = 0\} \tag{7} \]

By a “renewal” argument, which is different from the martingale approach, an integro-differential equation satisfied by the conditional probability of recovery is derived, based on which the probability of recovery is
formulated. In the case of exponential individual claim, the explicit expression for the probability of recovery is

\[ P(\hat{T} < \infty | R_0 = x) = 1 - \frac{\delta}{\lambda} \left( \frac{c}{\delta} \right)^{\frac{1}{\beta}} \exp\left( -\beta \left( \frac{x - c}{\delta} \right) t \right), \quad 0 \leq x \leq b, \]  

(8)

where $1/\beta$ is the mean of the exponential claim.

It should be mentioned that the probability of recovery is less than 1 when the individual claim amounts are exponentially distributed, whereas, for the classical risk model with constant dividend barrier and without debit interest rate, the probability of recovery is equal to 1.