CityU-Vanderbilt Workshop on Applied Mathematics

Date and Time: 15 May 2012, 9:00 am – 5:30 pm
16 May 2012, 9:30 am – 11:50 am
Venue: Room Y5-306, Academic 1, City University of Hong Kong

Programme at a Glance

15 May 2012 (Tuesday)

A.M. Session	Chair: Ding-Xuan Zhou	
9:00 am – 9:10 am	Roderick Wong (CityU)	Welcoming Remarks
9:10 am – 9:40 am	Edward Saff (Vanderbilt)	Asymptotics for Hessenberg Matrices for the Bergman Shift Operator
9:40 am – 10:10 am	Douglas Hardin (Vanderbilt)	Low Complexity Discrete Energy Problems with Varying Weights
10:10 am – 10:30 am	Coffee Break	
10:30 am – 11:00 am	Gieri Simonett (Vanderbilt)	On Well-posedness of Incompressible Two-phase Flows with Phase Transitions
11:00 am – 11:30 am	Dan Dai (CityU)	On Tronquée Solutions of the First Painlevé Hierarchy
11:30 am – 2:30 pm	Lunch Break	
P.M. Session	**Chair: Hui-Hui Dai**	
2:30 pm – 3:00 pm	Stephen Smale (CityU)	Mathematical Foundations of Immunology
3:00 pm – 3:30 pm	Alexander Powell (Vanderbilt)	Consistent Reconstruction and Random Polytopes
3:30 pm – 4:00 pm	Coffee Break	
4:00 pm – 4:30 pm	Ju-Yi Yen (Vanderbilt)	Option Prices in Terms of Probability Functions
4:30 pm – 5:00 pm	Qingshuo Song (CityU)	Outperformance Portfolio Optimization via the Equivalence of Pure and Randomized Hypothesis Testing
5:00 pm – 5:30 pm	Ding-Xuan Zhou (CityU)	Error Analysis and Sparsity of Some Learning Algorithms

16 May 2012 (Wednesday)

A.M. Session	Chair: Dan Dai	
9:30 am – 10:00 am	Glenn Webb (Vanderbilt)	Age Structured Population Dynamics and Applications to Biological Models
10:00 am – 10:30 am	Xiang Zhou (CityU)	Noise-induced Transition in Non-gradient Systems
10:30 am – 10:50 am	Coffee Break	
10:50 am – 11:20 am	Xuemei Chen (Vanderbilt)	Almost Sure Convergence Rate of the Kaczmarz Algorithm with Random Measurements
11:20 am – 11:50 am	Hui-Hui Dai (CityU)	An Analytical and Numerical Study on Wave Catching-up Phenomena in a Nonlinearly Elastic Composite Bar

Title and Abstracts of Presentation

15 May 2012 (Tuesdsay)

A.M. Session: 9:00 am – 11:30 am Chair: Ding-Xuan Zhou

9:00 am – 9:10 am Welcoming Remarks by Roderick Wong, City University of Hong Kong

9:10 am – 9:40 am Edward Saff, Vanderbilt University

Title: Asymptotics for Hessenberg Matrices for the Bergman Shift Operator

Abstract: For G a bounded Jordan domain in the complex plane we consider the Bergman polynomials $p_n(z)$ that are orthonormal with respect to area measure over G. They are uniquely defined by entries of an infinite upper Hessenberg matrix M, which represents the Bergman shift operator of G. We analyze the relationship between M and the Toeplitz matrix with symbol the normalized conformal map of the exterior of the unit circle on the complement of the closure of G. As an application we describe an algorithm for recovering the shape of G from its area moments.

9:40 am – 10:10 am Douglas Hardin, Vanderbilt University

Title: Low Complexity Discrete Energy Problems with Varying Weights

Abstract: We consider asymptotic (as $N \to \infty$) geometrical properties of N-point configurations $\{x_i\}_{i=1}^N$ on a d-rectifiable set A that minimize a *weighted Riesz s-energy* functional of the form

$$\sum_{i \neq j} \frac{w_N(x_i, x_j)}{|x_i - x_j|^s}$$

for a given 'weight' function w_N on $A \times A$ and a parameter $s > 0$. In previous work, we described the asymptotic distribution for such problems when the weight w was a 'CPD' weight not depending on N. We extend these results to the case of N-dependent weights. In particular, we consider weights that lead to low complexity energy calculations.

10:10 am – 10:30 am Coffee Break

10:30 am – 11:00 am Gieri Simonett, Vanderbilt University

Title: On Well-posedness of Incompressible Two-phase Flows with Phase Transitions

Abstract: A model for incompressible two-phase flows with phase transitions is derived from basic principles and shown to be thermodynamically consistent in the sense that the total energy is conserved and the total entropy is nondecreasing. Local well-posedness is obtained by means of the technique of maximal L_p-regularity in the case of equal densities. This way we obtain a local semiflow on a well-defined nonlinear state manifold. The equilibria of the system in absence of external forces are identified and it is shown that the negative total entropy is a strict Ljapunov functional for the system. If a solution does not develop singularities, it is shown that it exists globally in time, its orbit is relatively compact, and its limit set is nonempty and contained in the set of equilibria.

11:00 am – 11:30 am Dan Dai, City University of Hong Kong

Title: On Tronquée Solutions of the First Painlevé Hierarchy

Abstract: It is well known that the first Painlevé equation admits solutions characterized by divergent asymptotic expansions near infinity in specified sectors of the complex plane. In this talk, we show that such solutions exist for higher order analogues of the first Painlevé equation (the first Painlevé hierarchy) as well.

11:30 am – 2:30 pm Lunch Break

P. M. Session: 2:30 pm – 5:30 pm Chair: Hui-Hui Dai

2:30 pm – 3:00 pm Stephen Smale, City University of Hong Kong

Title: Mathematical Foundations of Immunology

Abstract: We will discuss String Kernels, Binding, and Genes.

3:00 pm – 3:30 pm Alexander Powell, Vanderbilt University

Title: Consistent Reconstruction and Random Polytopes

Abstract: Consistent reconstruction is a linear programming technique for reconstructing a signal from a set of quantized linear measurements. We prove new mean squared error bounds (MSE) for consistent reconstruction in the setting of random frames and under the uniform quantization noise model. In particular, we prove that the mean squared error for consistent reconstruction is of the optimal order $1/N^2$ where N is the number of measurements, and we prove bounds on the associated dimension dependent constant. For comparison, in the important case of unit-norm tight frames with linear reconstruction (instead of consistent reconstruction) the mean squared error only satisfies a weaker bound of order $1/N$. Our results require a mathematical analysis of random polytopes generated by random hyperplanes and of associated coverage processes on the sphere. This is joint work with Tyler Whitehouse.

3:30 pm – 4:00 pm Coffee Break

4:00 pm – 4:30 pm Ju-Yi Yen, Vanderbilt University

Title: Option Prices in Terms of Probability Functions

Abstract: The Black-Scholes model is an important starting point for studying financial derivatives. In the Black-Scholes formula, the evolution of prices of a risky asset is described by an exponential martingale associated to a Brownian motion and, as a consequence, the Black-Scholes function is increasing and bounded and can be written as a distribution function. We shall explore the connection between Black-Scholes type functions and their distribution functions. We study the distribution function in terms of the last passage times, and extend the underlying martingale beyond the Brownian framework. Explicit examples of computations of these laws are given.

4:30 pm – 5:00 pm Qingshuo Song, City University of Hong Kong

Title: Outperformance Portfolio Optimization via the Equivalence of Pure and Randomized Hypothesis Testing

Abstract: We study the portfolio problem of maximizing the outperformance probability over a random benchmark through dynamic trading with a fixed initial capital. Under a general incomplete market framework, this stochastic control problem can be formulated as a composite pure hypothesis testing problem. We analyze the connection between this pure testing problem and its randomized counterpart, and from latter we derive a dual representation for the maximal outperformance probability. Moreover, in a complete market setting, we provide a closed-form solution to the problem of beating a leveraged exchange traded fund. For a general benchmark under an incomplete stochastic factor model, we provide the Hamilton-Jacobi-Bellman PDE characterization for the maximal outperformance probability. This is a joint work with Leung Tim and Jie Yang.

5:00 pm – 5:30 pm Ding-Xuan Zhou, City University of Hong Kong

Title: Error Analysis and Sparsity of Some Learning Algorithms

Abstract: Learning theory studies learning function relations or data structures from samples. In this talk we shall discuss a class of kernel-based learning algorithms generated by regularization schemes based on empirical features. The algorithms produce sparse approximations for regression.

16 May 2012 (Wednesday)

A.M. Session: 9:00 am – 11:50 am Chair: Dan Dai

9:30 am – 10:00 am Glenn Webb, Vanderbilt University

Title: Age Structured Population Dynamics and Applications to Biological Models

Abstract: Individual age structure arises in many models in biology and medicine. A general theory of age structured population models will be presented. Applications will be given to models of epidemics structured by infection age and models of cell proliferation structured by cell cycle age.

10:00 am – 10:30 am Xiang Zhou, City University of Hong Kong

Title: Noise-induced Transition in Non-gradient Systems

Abstract: Noise-induced transitions occur in many stochastic dynamical systems with multiple metastable states, and it is important to understand relevant physical/chemical/biological processes. Instead of well-developed progress for gradient systems driven by energy potential, I will address the transition problems for non-gradient dynamics, which has potential applications to fluid and biology which lack of energy landscape. This talk introduces an adaptive version of minimum action method for small noise diffusion processes, and discusses the transition pathways for stochastic Lorenz systems and Kuramoto-Sivashinsky SPDE. I will also stress how to understand subcritical instability in physics and fluid dynamics from perspective of noise-induced transition.

10:30 am – 10:50 am Coffee Break

10:50 am – 11:20 am Xuemei Chen, Vanderbilt University

Title: Almost Sure Convergence Rate of the Kaczmarz Algorithm with Random Measurements

Abstract: The Kaczmarz algorithm is an iterative method for reconstructing a signal $x \in \mathbb{R}^d$ from an overcomplete collection of linear measurements $y_n = \langle x, \varphi_n \rangle$, $n \geq 1$. We prove quantitative bounds on the rate of almost sure exponential convergence in the Kaczmarz algorithm for suitable classes of random measurement vectors $\langle \varphi_n \rangle_{n=1}^{\infty} \subset \mathbb{R}^d$. Refined convergence results are given for the special case when each φ_n has i.i.d. Gaussian entries and, more generally, when each $\varphi_n / \|\varphi_n\|$ is uniformly distributed on $\mathbb{S}^{d-1}$.

11:20 am – 11:50 am Hui-Hui Dai, City University of Hong Kong

Title: An Analytical and Numerical Study on Wave Catching-up Phenomena in a Nonlinearly Elastic Composite Bar

Abstract: Cracking induced by tensile wave at the free surface of an impacted target is an important issue in impact-resistant design. Here, we explore to use material nonlinearity to undermine the strength of the tensile wave. More specifically, we consider waves in a two-layer nonlinearly elastic composite bar induced by a compressive impact. Multiple reflections cause a tensile wave being transmitted into the second layer. The phenomenon that the tensile wave catches the first transmitted compressive wave is studied analytically and numerically. It is shown that there are two wave patterns in which catching-up phenomena can happen. Besides a general mathematical theory, asymptotic solutions are also constructed, which provide the insights on the requirement of the constitutive relation, the time and place at which the catching takes place, and how the initial impact, material and geometric parameters influence the solutions. Numerical solutions are also obtained, confirming the validity of the analytical results. This is a joint work with Z. Chen, S.-J. Huang, D.-X. Kong, J. Jiang and Y. Xu.