

Undergraduate Textbook in Mathematics
Hong Kong Mathematical Society

香港數學學會大學數學教科書系列

1

Calculus

Rigor, Concision, Clarity

Jishan Hu
Jian-Shu Li
Wei-Ping Li
Min Yan



Hong Kong Mathematical Society
Undergraduate Textbook in Mathematics

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Preface

Publishing undergraduate and graduate textbooks in Mathematics is a recent initiative of Hong Kong Mathematical Society (HKMS). The rationale behind this project is to encourage and support colleagues and scholars to publish their own teaching materials so that they can be used by more students associated with our Society. Our aim is to offer high quality textbooks at reasonable prices and to increase the visibility of HKMS to others in the field.

We are delighted to have the great support from our colleagues Jishan Hu, Jian-Shu Li, Wei-Ping Li and Min Yan of the Hong Kong University of Science and Technology (HKUST), who have contributed this present book: the first book of our Undergraduate Textbook Series. The book has been used for quite some years for undergraduate students at HKUST as their textbook. It is expected that students from other universities will find the book useful for their study. It may also serve as supplementary materials for undergraduate students.

With the success of launching this meaningful project, HKMS will continue to publish high-quality and original textbooks in Mathematics on both undergraduate and graduate levels.

We welcome colleagues to share their lecture notes with our students. HKMS will provide necessary support for copy editing. For any publication proposal or questions, please contact us at hkms@www.hkms.org.hk.

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is a text that

- has been used by the authors to teach Calculus courses at Hong Kong University of Science and Technology in the past ten years;
- shows the main ideas and basic applications of Calculus in a step-by-step manner;
- contains many useful Exercise questions which are found useful by students.

It is a useful textbook or reference for students in Science and Engineering.

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