RBF Multiscale Collocation for Second Order Elliptic Boundary Value Problems

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In this talk, we discuss multiscale radial basis function collocation methods for solving elliptic partial differential equations on bounded domains. The approximate solution is constructed in a multi-level fashion, each level using compactly supported radial basis functions of smaller scale on an increasingly fine mesh. On each level, standard symmetric collocation is employed. A convergence theory is given, which builds on recent theoretical advances for multiscale approximation using compactly supported radial basis functions.

If time permits, we also discuss the condition numbers of the arising systems as well as the effect of simple, diagonal preconditioners. In particular, we present results that prove previous numerical observations made by Fasshauer [1]. More than a decade ago, he observed:

- There is no convergence in the stationary setting, i.e. if the support radius at a given level is chosen proportional to the mesh norm of that level.
- There is convergence, if the support radii go slower to zero than the mesh norms.
- In contrast to pure interpolation, even in the stationary setting, the condition numbers of the collocation matrices depend on the level.
- In the stationary case, a simple preconditioning $PAP = Py$ with a diagonal matrix P leads to a level-independent condition number.
- This preconditioning technique does not lead to a level-independent condition number in the non-stationary setting, where convergence occurs.

References

[1] Gregory E. Fasshauer, *Solving Differential Equations with Radial Basis Functions: Multilevel Methods and Smoothing*, Adv. Comput. Math. **11** (1999), no. 2–3, 139–159, Radial basis functions and their applications.