Boundary Effects, Norm Minimizing Extension and the Polyharmonic Dirichlet Problem

Thomas Hangelbroek

University of Hawai'i at Mānoa, USA

hangelbr@math.hawaii.edu

The presence of a boundary in kernel approximation poses a number of computational challenges. One such challenge is very well-understood for surface spline approximation: Johnson [**?**] showed that on bounded domains approximation is saturated at a much lower rate than in the boundary-free case. At the same time, precise rates of approximation for many standard function classes have been unknown in this setting. Consequently, the estimate for the saturation order is greater than the best known approximation order for smooth functions.

For fairly general domains in $\mathbb{R}^d$ (compact with smooth boundaries), we present an approximation scheme for surface spline approximation that delivers L_p, $(1 \leq p \leq \infty)$ approximation orders for well known smoothness spaces. In particular, Johnson's saturation order estimate is tight. Furthermore, for sets of centers having extra density near the boundary (violating an assumption in [**?**]) the increased free-space convergence rate can be achieved. Connections between this scheme, norm minimizing extension of smooth functions and certain elliptic boundary value problems will be discussed.

References

[1] Johnson, M. J, *A bound on the approximation order of surface splines*, Constr. Approx. 14 (1998), no. 3, 429438.