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# The Tacnode Riemann-Hilbert Problem

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A main topic in random matrix theory and related models is the study of critical phenomena and phase transitions. This corresponds to situations where gaps appear in the limiting supports of eigenvalues when parameters in the model change. A detailed local analysis can in certain situations be done with special functions of the Painlevé type.

The tacnode is a critical phenomenon that appears in the context of non-intersecting Brownian motions with prescribed starting and ending positions. One may create a situation in which the Brownian motions fill out two tangent ellipses in the time-space plane.

Local correlations at the tacnode have been described in two different ways. On the one hand there are formulas with integrals of Airy resolvent functions, due to Adler, Ferrari, Johansson, Van Moerbeke, and Vető [1], [4], [5]. On the other hand there exists a formulation in terms of a Riemann-Hilbert problem due to Delvaux, Zhang and the speaker [3]. I will discuss these two formulations and the relation between them that was recently found by Delvaux [2].

### References

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