Asymptotic Behaviour of Zeros of Exceptional Jacobi and Laguerre Polynomials

Francisco Marcellán

Universidad Carlos III de Madrid, Spain

pacomarc@ing.uc3m.es

In this presentation we analyze the location and asymptotic behaviour, for large $n,$ of the zeros of exceptional Jacobi and Laguerre polynomials. These families of polynomials have been introduced in [2] and attracted an increasing attention in the last three years not only from a mathematical point of view (their relation with Sturm-Liouville problems and and Darboux transformations of second order linear differential operators) but also because of their applications in integrable systems (exactly solvable quantum mechanical problems), entropy measures in quantum information theory, rational extensions of Morse and Kepler-Coulomb problems, among others.

More recently, analytic properties of such zeros have been studied (see [3]). The zeros of exceptional polynomials fall into two classes: the *regular zeros*, which lie in the interval of orthogonality and the *exceptional zeros*, which lie outside that interval. We show that the regular zeros have two interlacing properties: one is the natural interlacing between consecutive polynomials as a consequence of their Sturm-Liouville character, while the other one shows interlacing between the zeros of exceptional and classical polynomials.

A generalization of the classical Heine-Mehler formula is provided for the exceptional polynomials, which allows to derive the asymptotic behaviour of their regular zeros. We also describe the location and the asymptotic behaviour of the *exceptional zeros*, which converge for large n to fixed values.

Finally, we discuss the monotonicity of the zeros of exceptional Laguerre polynomials as well an electrostatic interpretation of them, according to the analysis done in [1] in the Jacobi case.

This is a joint work with D. Gómez-Ullate (Universidad Complutense Madrid, Spain) and R. Milson (Dalhousie University, Canada).

References

[1] D. K. Dimitrov, Y-C. Lui, *Monotonicity, interlacing and electrostatic interpretation of zeros of exceptional Jacobi polynomials.* Preprint

[2] D. Gómez-Ullate, N. Kamran, and R. Milson, *An extended class of orthogonal polynomials defined by a Sturm-Liouville problem*, J. Math. Anal. Appl. **359** (2009) 352–367.

[3] D. Gómez-Ullate, F. Marcellán, and R. Milson, *Asymptotic and interlacing properties of zeros of exceptional Jacobi and Laguerre polynomials*, J. Math. Anal. Appl. **399** (2013) 480–495.