S-curves and (Non-hermitian) Orthogonal Polynomials

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Consider a sequence of polynomials \((P_n)\) satisfying the (non-hermitian) complex orthogonality

\[
\int_{\Gamma} z^j P_n(z) e^{-nV(z)} dz = 0, \quad j = 0, \ldots, n - 1,
\]

where \(V\) is a fixed polynomial and the integration is on an unbounded simple contour \(\Gamma\) in \(\mathbb{C}\) ending up at \(\infty\) in both directions and such that \(\text{Re}V(z) \to +\infty\), as \(z \to \infty\) in \(\Gamma\).

If the polynomial \(V\) is real and \(\Gamma = \mathbb{R}\), the zeroes of the \(P_n\)'s are also real and their limiting distribution can be characterized in terms of an equilibrium problem with external field on the real line. In contrast, if \(V\) is no longer real we have a lot of freedom in choosing the contour \(\Gamma\), and this freedom is reflected in the behavior of the zeroes of the polynomials \(P_n\)'s.

Gonchar and Rakhmanov [1] characterized the limiting distribution of these zeroes, conditioned to the existence of a curve \(\Gamma\) with a certain symmetry property - the so called S-property - over which we can compute the integrals above.

Based on recent works [2,3], we will discuss the existence of this curve \(\Gamma\) and its characterization.

This is a joint work with Arno Kuijlaars.

References