
Empirical Invariance in Stock Market and Related Problems

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We consider companies in S&P500 list which varies slightly each year. The intraday transactions data (trades and quotes) are downloaded from the Trade and Quote (TAQ) database in the Wharton Research Data Services. For easier analysis the stock price, cumulative volume, and number of transactions (frequency) are stored for each 30 seconds. The returns are considered first at five-minute and one-minute intervals for each year. For each stock the empirical distribution of the waiting time for a success, i.e. to hit a certain percentile (e.g. the upper or lower ten percentile) of the returns, is considered. Most of the empirical distributions are close to each other across different stocks and different years under two comparison criteria, ROC area and Kolmogorov-Smirnov distance. This is an empirical invariance. The invariance holds for shorter time periods and different unit time intervals. The overlapping of the time points corresponding to successes for different stocks, the probability of the zero waiting time for a success, the outliers, empirical entropy, other percentiles are also considered. It is worth mentioning that the stock prices of the outliers have financial meaning. Invariances are also found for the empirical distributions of the waiting time for hitting the upper ten percentile of the cumulated volume process and the frequency process. The positive correlation among these three processes during the highly volatile periods is observed empirically.

Mathematically we prove that the empirical distributions of the classical occupancy problem converge to a geometric distribution a.s. This result is applicable to the case when the return follows a Levy process. One may regard this as a law of large numbers for the corresponding empirical processes, although the limit is already an invariance. Simulation shows that the right scaling is square root of n . However, some types of Kolmogorov theorem and Donsker's theorem are yet to be proved.

When the stock price following an exponential fractional Brownian motion with index differs from one half, it has a different invariance distribution empirically. Note that the empirical invariance found in the market is quite far away from the geometric distribution as well as the empirical distributions obtained from the fractional Brownian motion.

The financial and mathematical implications and questions will be discussed. This is an ongoing research.