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Abstracts

ABSTRACTS OF TALKS

Explicit Error Bounds for Asymptotic Solutions of Linear Difference Equations

Lihua CAO
City University of Hong Kong

In this talk,we consider the asymptotic solutions of linear differdnce equations constructed by R. Wong and H. Li. By studying a compatison equation, we obtain explicit and numerically computable error bounds of the asymptotic expansions of these solutions.

Painleve type asymptotics in orthogonal polynomials

Dan DAI
City University of Hong Kong

It has been realized that Painleve transcendents play an important role in asymptotics of orthogonal polynomials. In this talk, we will review some results related to this topic and present something new.

Analysis of a perturbed solution for the one-dimensional nonlinear Beam equation

X. H. Jiang & L. H. shi
Beijing University of Chemical Technology

We investigate the initial-boundary value problem for the one-dimensional nonlinear beam equation with fixed ends. A first-term approximate solution is constructed via the method of multiple scales and the error of the approximate solution is estimated by using energy method combining with nonlinear Gronwall's inequality.

Keywords: Beam equation; initial-boundary value problem; the method of multiple scales

Uniform Asymptotic Expansions of the Tricomi-Carlitz Polynomials

Kei Fung LEE
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The Tricomi-Carlitz polynomials satisfy the second-order linear difference equation

$$(n+1)f_{n+1}^{(\alpha)}(x) - (n+\alpha)\,x\,f_n^{(\alpha)}(x) + f_{n-1}^{(\alpha)}(x) = 0, \qquad n \geq 1,$$

with initial values $f_0^{(\alpha)}(x) = 1$ and $f_1^{(\alpha)}(x) = \alpha x$, where x is a real variable and α is a positive parameter. An asymptotic expansion is derived for these polynomials by using different approaches, including the turning-point theory for three-term recurrence relations developed by Wang and Wong [Numer. Math. **91**(2002) and **94**(2003)], method of steepest descent. Comparisons of these results are made and numerical results are attached. The result holds uniformly in regions containing the critical values $x = \pm 2/\sqrt{\nu}$, where $\nu = n + 2\alpha - 1/2$.

Asymptotic Approximations of Three Term Recurrence Relations with Applications

Yutian LI
City University of Hong Kong

In this talk, we will discuss the recent development in studying the asymptotics of three term-recurrence relations (TTRR). Since any sequence of polynomials orthogonal on the real line (OPRL) satisfies a three-term recurrence relation, we may study the asymptotics of these polynomials via the three term recurrence relation. One can use these results to get the point-wise (fixed variable) asymptotics or scaled-variable asymptotics. As to the uniform asymptoics, we will discuss the turning point theory founded by Z. Wang and R. Wong. Examples include both classical and discrete polynomials, such as Jacobi, Laguerre, Hermite, Charlier, Mexiner, Tricomi-Calitz polynomials. Applications of TTRR to the asymptotics of the Landau constants and Apéry's sequence will also be mentioned.

Global Asymptotic for the Hahn polynomials

Yu LIN
City University of Hong Kong (Shenzhen) Research Institute

The Hahn polynomials are a very important class of the discrete orthogonal polynomials arising in various problems of physics, mathematics and engineering sciences. It has been known that the existing asymptotic methods for integrals and differential equations are not applicable in the case of the Hahn polynomials. In this talk, we investigate the asymptotics of the Hahn polynomials as $n \to \infty$, when the ratio of the parameters n/N is a constant $c \in (0, 1)$. Global asymptotic formulas are obtained in the complex z-plane when n goes to infinity. Our approach is based on a modified version of the steepest- descent method for Riemann-Hilbert problems introduced by Deift and Zhou.

A Problem Raised by D.J. Benney

Chunhua OU
University of Newfoundland

It is well-known that the naive asymptotic analysis of perturbed Duffing equation can result in the occurrence of secular terms so that the asymptotic expansions are not valid for time t tending to infinity. Whether or not this phenomenon appear in one class of partial differential equations was a problem raised by Prof. R. Wong several years ago, which was originally from Prof. D.J. Benney. In this talk, under some conditions on the initial data, non-existence of secular terms (non-uniformity) is investigated via some case studies including Burgers equation. Rigorous asymptotic expansions are provided for three perturbed partial differential equations in unbounded domains.

Error bound for the Airy-type integral transform

Jianhui PAN
City University of Hong Kong

The purpose of this talk is to discuss about the error analysis of asymptotic expansions of integral transforms. Particular attention is paid to the integral transform of Airy type. The error bound is found by exploiting the reformulation of the method involving rational function given by Daalhuis and Temme. The method will be demonstrated by the Hermite polynomial example, and an explicit error bound result will be given at the end.

Asymptotic analysis in biological mathematics – calculating dynamic threshold of periodic system with time delay

Xiangsheng WANG
York University

One of the most important concepts in biological mathematics is the basic reproduction number which characterizes a threshold dynamic in ecology or epidemiology: if this index is greater than one, then the population in ecologic model will not be extinct, or the disease in epidemic model will spread. From mathematical points of view, it is related to the principle spectrum of the solution operator in a dynamical system, namely, the solution will tend to zero if and only if the basic reproduction number is less than one. This means that we have to study long-time asymptotic behavior of the solution to the dynamic system. However, for a general periodic system with time delay, it seems impossible to calculate the threshold explicitly in terms of system parameters. Here, we will make two different attempts to address this problem: one is deriving an asymptotic formula for the dynamic threshold with the assumption that some parameters in the system are extremely large or small; the other method is discretizing the differential equations and then conducting asymptotic analysis on the corresponding difference equations. This is an ongoing joint work with Jianhong Wu.

Parametrix and Painlevé functions involved in an on-going investigation

Yu-Qiu ZHAO
Sun Yat-sen University

I will talk on the local parametrix associated with a singularly perturbed Laguerre weight with a varying parameter, and the possible construction using a Painlevé transcendent. These would be parts of an on-going investigation with Dan Dai.