Nonlinear Approximation and (Deep) ReLU Networks

INGRID DAUBECHIES, RON DEVORE, SIMON FOUCART*, BORIS HANIN, GUERGANA PETROVA
Department of Mathematics, Texas A&M University, USA
Email: `foucart@tamu.edu`

This talk is concerned with the approximation power (sometimes called the expressive power) of deep neural networks, an active research area currently producing many interesting papers. The most common results found in the literature prove that neural networks approximate functions with classical smoothness to the same accuracy as classical methods, e.g. approximation by polynomials or piecewise polynomials on prescribed partitions. However, approximation by neural networks depending on n parameters is a form of nonlinear approximation and as such should exhibit the increased efficiency of nonlinear approximation methods. The present work shows that this is indeed the case.

Furthermore, the performance of neural networks in targeted applications such as machine learning indicate that they actually possess even greater approximation power than traditional methods of nonlinear approximation, such as free knot splines or n-term approximation from a dictionary. The main results of our work again shows that this is indeed the case. To do so, we exhibit large classes of functions which can be efficiently captured by neural networks where classical nonlinear methods fall short of the task. Our work purposefully limits itself to studying the approximation of univariate functions by ReLU networks. Many generalizations to functions of several variables and other activation functions can be envisioned. However, even in this simplest of settings considered here, a theory that completely quantifies the approximation power of neural networks is still lacking.