Mathematical Assessment of PWM Control of A Induction Motor

Z. A. Aziz, C. S. Bohun, K. W. Chung,
H. H. Dai, Y. H. Hao, D. Ho, H. X. Huang,
F. F. Wang, J. Wang, Y. B. Wang

December 11, 2009

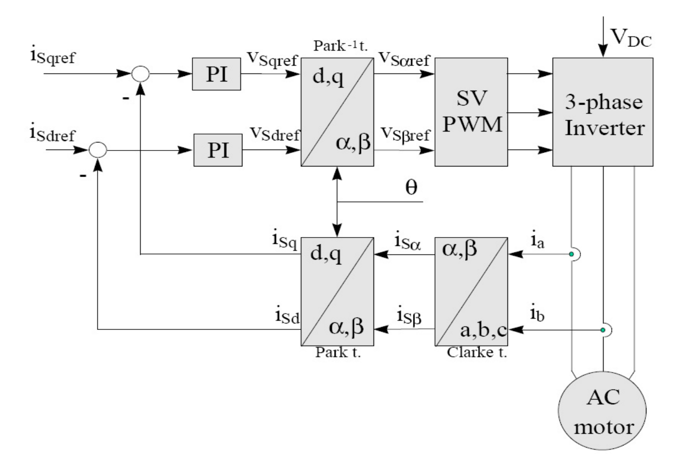
1. Motivations
2. Simple single coil AC motor model
3. Three Phase Induction Motor Model and Simulation
4. Sensorless control

Motivations

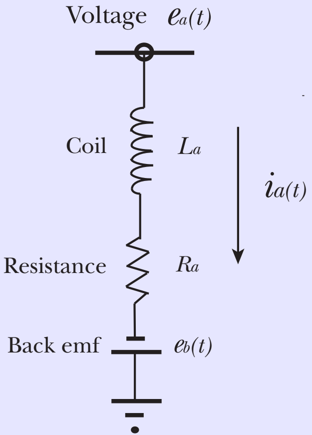
This project aims to develop PWM based schemes to control single phase AC induction to operate at different rotational speed with optimized motor performance in terms of:

- High torque output, particularly during start-up;
- High energy efficiency;
- Shortest spin-up time.

Basic scheme of field orientated control for 3-phase AC-motors



Simple single coil AC motor model



$$e_a(t) = L_a\, i_a'(t) + R_a\, i_a(t) + e_b(t), \qquad (1)$$

$$J\,\omega'(t) = \tau_m(t) - \tau_L, \qquad (2)$$

$$e_b(t) = K_b\, \omega(t), \qquad (3)$$

$$\tau_m(t) = K_i\, i_a(t). \qquad (4)$$

Governing equation and general solution

From (1)-(4) and by eliminating $\omega(t)$, $e_b(t)$ and $\tau_m(t)$, we obtain

$$i_a''(t) + \frac{R_a}{L_a} i_a'(t) + \frac{K_i K_b}{L_a J} i_a(t) = \frac{e_a'(t)}{L_a} - \frac{K_b \tau_L}{L_a J} = f(t), \tag{5}$$

The general solution of equation (5) is

$$i_a(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + \frac{e^{\lambda_2 t}}{\lambda_2 - \lambda_1} \int_0^t e^{-\lambda_2 s} f(s) ds - \frac{e^{\lambda_1 t}}{\lambda_2 - \lambda_1} \int_0^t e^{-\lambda_1 s} f(s) ds. \tag{6}$$

At time $t = 0$, we have

$$i_a(0) = 0, \quad i_a'(0) = C_1 \lambda_1 + C_2 \lambda_2. \tag{7}$$

Control objectives

- Find the optimized voltage input $e_a^o(t)$, s.t.,

$$\int_0^T (i_a^o(t) - \bar{i}_a)^2 dt = \min_{e_a(t)} \int_0^T (i_a(t) - \bar{i}_a)^2 dt. \qquad (8)$$

- The following constraint condition is satisfied

$$\int_0^T e_a(t) i_a(t) dt \leq \alpha_0. \qquad (9)$$

We consider the Fourier series expansion form of $e_a(t)$:

$$e_a(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(\frac{2\pi n}{t_p} t) + b_n \sin(\frac{2\pi n}{t_p} t)]. \qquad (10)$$

Simulation results (1)

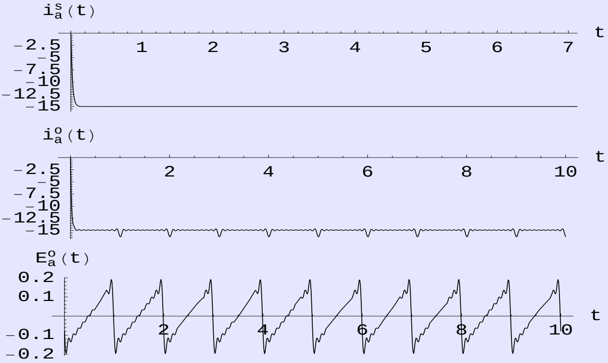


Fig 2. Simulation results with $R_a = 0.1\Omega$, $L_a = 10^{-4}$ H, $J = 9 \times 10^{-5}$ Kg $\cdot\mathrm{m}^2$, $K_i = K_b = 0.02$, $\tau_L = 0.3$ N·m.

Simulation results (2)

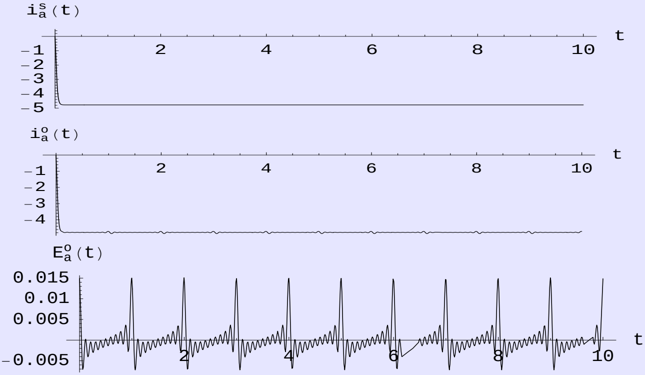
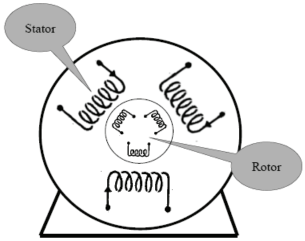


Fig 3. Simulation results with $R_a = 0.142\Omega$, $L_a = 1.1 \times 10^{-3}$ H, $J = 0.088$ Kg $\cdot \mathrm{m}^2$, $K_i = K_b = 0.628$, $\tau_L = 3$ N·m.

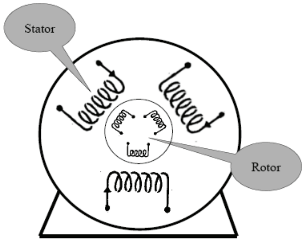


1. Current of stator and rotor in d- and q-coordinate: $i_{dS}, i_{qS}, i_{dR}, i_{qR}$;
2. Flux-linkage of stator and rotor in d- and q-coordinate: $\lambda_{dS}, \lambda_{qS}, \lambda_{dR}, \lambda_{qR}$;
3. Resistance of stator and rotor: r_S, r_R;
4. Self-inductance of stator and rotor: L_S, L_R;
5. Magnetizing inductance: M.

Flux-current relationships are

$$\begin{bmatrix}\lambda_{dS}\\ \lambda_{dR}\end{bmatrix} = \begin{bmatrix}L_S & M\\ M & L_R\end{bmatrix}\begin{bmatrix}i_{dS}\\ i_{dR}\end{bmatrix}$$

$$\begin{bmatrix}\lambda_{qS}\\ \lambda_{qR}\end{bmatrix} = \begin{bmatrix}L_S & M\\ M & L_R\end{bmatrix}\begin{bmatrix}i_{qS}\\ i_{qR}\end{bmatrix}$$



The voltage equations are

$$\begin{cases} v_{dS} = \dfrac{d\lambda_{dS}}{dt} - \omega\lambda_{qS} + r_S i_{dS}, \\ v_{qS} = \dfrac{d\lambda_{qS}}{dt} + \omega\lambda_{dS} + r_S i_{qS}, \\ 0 = \dfrac{d\lambda_{dR}}{dt} - \omega_s\lambda_{qR} + r_R i_{dR}, \\ 0 = \dfrac{d\lambda_{qR}}{dt} + \omega_s\lambda_{dR} + r_R i_{qR}. \end{cases}$$

The balance of the torques is

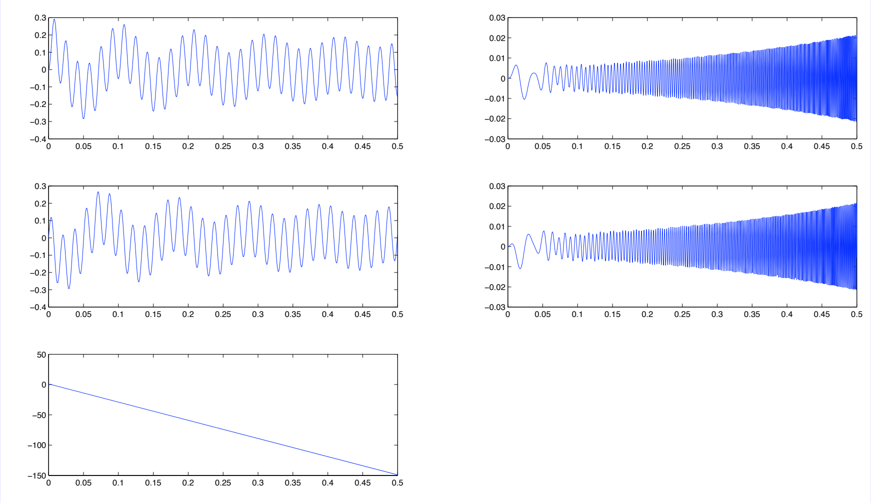
$$T^e = J\frac{d\omega_m}{dt} + T_L,$$

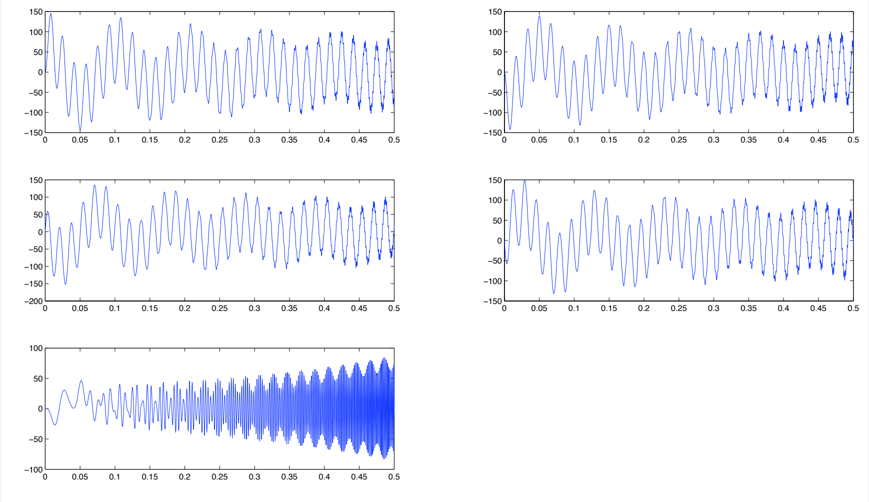
$$T^e = \frac{3}{2}p(\lambda_{dS}i_{qS} - \lambda_{qS}i_{dS}),$$

$$\omega_s = \omega - p\omega_m.$$

$$y_1 = \lambda_{dS}, \quad y_2 = \lambda_{dR}, \quad y_3 = \lambda_{qS}, \quad y_4 = \lambda_{qR}, \quad y_5 = \omega_m. \tag{11}$$

$$\begin{cases} \dfrac{dy_1}{dt} = -c_1 y_1 - c_2 y_2 + \omega y_3 + v_{dS}, \\ \dfrac{dy_2}{dt} = -c_3 y_1 - c_4 y_2 + \omega y_4 - p y_4 y_5, \\ \dfrac{dy_3}{dt} = -\omega y_1 - c_1 y_3 - c_2 y_4 + v_{qS}, \\ \dfrac{dy_4}{dt} = -\omega y_2 - c_3 y_3 - c_4 y_4 + p y_2 y_5, \\ \dfrac{dy_5}{dt} = c_5 y_1 y_4 - c_5 y_2 y_3 + c_6. \end{cases} \tag{12}$$


0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5
0.03
0.02
0.01
0
−0.01
−0.02
−0.03
50
0
−50
−100
−150


150
100
50
0
−50
−100
−150
−200
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5

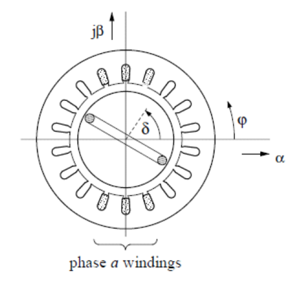
Sensorless control

Main ingredients:

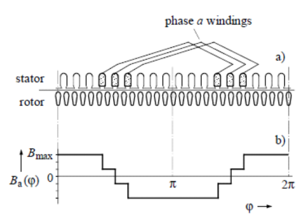
- motion of the rotor induces a current back into the voltage of the stator
- this signal can be recovered by sampling the stator voltage
- the phase of this signal depends on the position of the rotor
- this signal can be used in place of the position detector circuitry that industry would like to remove

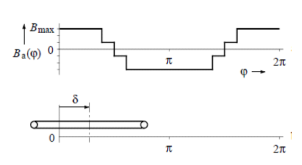
Position dependent mutual inductance

Single applied stator voltage



Induced magnetic field



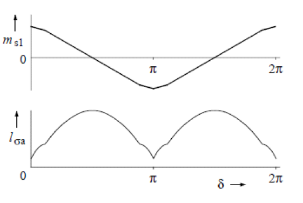


$$m_{s1}(\delta) = \int_{\delta-\pi/2}^{\delta+\pi/2} l B_a(\theta)\, d\theta$$

Induced voltage in the stator

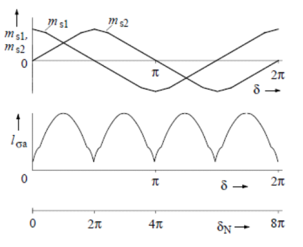
Induced voltage $u_{sa} = l_{\sigma a}\frac{di_a}{d\tau}$

One rotor bar



$$l_{\sigma a} = l_s\left(1 - \frac{m_{a1}^2}{l_s l_1}\right)$$

Two rotor bars



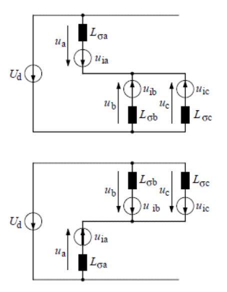
$$l_{\sigma a} = l_s\left(1 - \frac{m_{a1}^2 + m_{a2}^2}{l_s l_1}\right)$$

$$l_1 = l_2,\ m_{12} = 0$$

Switching of the voltage to the stator

The voltage differences are functions of these position dependent mutual inductances

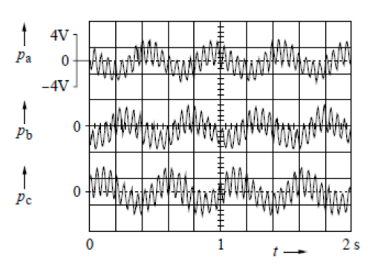
$$u_\sigma = u_a + u_b + u_c, \quad u_\sigma^{(1)} - u_\sigma^{(4)} = 2U_d \frac{l_{\sigma a} l_{\sigma b} + l_{\sigma a} l_{\sigma c} - 2 l_{\sigma b} l_{\sigma c}}{l_{\sigma a} l_{\sigma b} + l_{\sigma a} l_{\sigma c} + l_{\sigma b} l_{\sigma c}}$$

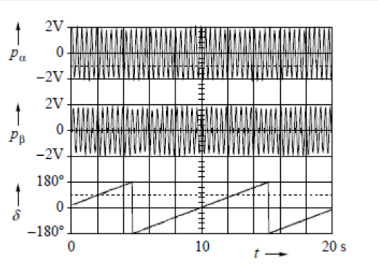


Construction of the position estimate

$$p_a = u_\sigma^{(1)} - u_\sigma^{(4)}, \qquad p_b = u_\sigma^{(3)} - u_\sigma^{(6)}, \qquad p_c = u_\sigma^{(5)} - u_\sigma^{(2)}$$

$$p_\alpha + j p_\beta = p_a + e^{2\pi j/3} p_b + e^{-2\pi j/3} p_c$$





References

- M. W. Degner and R. D. Lorenz, Using Multiple Saliencies for the Estimation of Flux, Position and Velocity in AC Machines, *IEEE Transactions on Industry Applications*, Vol, 34, No. 5, Sept/Oct. 1998, PP. 1097-1104.
- J.-I. Ha and S.-K. Sul, Sensorless Field-Oriented Control of an Induction Machine by High-Frequency Signal Injection, *IEEE Transactions on Industry Applications*, Vol, 35, No. 1, Jan/Feb. 1999, PP. 45-51.
- J. Holtz, Sensorless Control of Induction Motor Drives, *Proceedings of the IEEE*, Vol. 90, No. 8, Aug. 2002, pp. 1359-1394.
- M. Linke, R. Kennel, and J. Holtz, Sensorless Speed and Position Control of Permanent Magnet Synchronous Machines, *IECON, 28th Annual Conf. of the IEEE Industrial Electronics Society*, Sevilla/Spain, 2002, on CD ROM.
- M. Schroedl, Sensorless Control of AC Machines at Low Speed and Standstill based on the Inform Method, *IEEE Industry Applications Society Annual Meeting*, Pittsburgh, Sept. 30 - Oct. 4, 1996, pp. 270-277.

Thank you!