Smoke Propagation Problem

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Introduction

- Aim:
  To build a simple mathematical model in order to assess both the CFD models and empirical models for soot deposition, and hence to improve visibility prediction.
- Physical Problem:
  Point source fire on floor causes forced thermal convection which transports diffusing smoke. Soot particles mostly move by convection and diffusion, and they can deposit on the ceiling.
  We consider the dimensionless 2-D 'steady state' problem in $y < 0$, in which soot layer grows at the ceiling $y = 0$, but flow and particle concentration are independent of time $(u(x, y), v(x, y), c(x, y))$. (Hope to do the 3-D unsteady problem later.)

Introduction

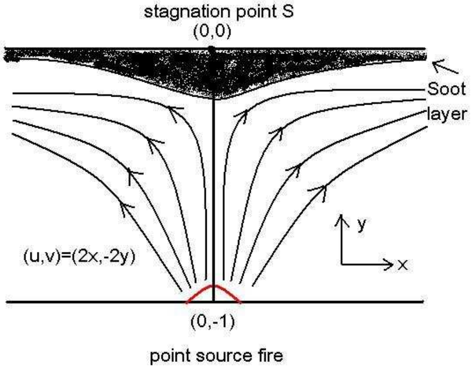


Figure: Illustrative Picture

Governing Equations

We should find fluid velocity from a thermal convection model. But here we will simply assume this leads to a locally isothermal flow (u, v) near the stagnation point on the ceiling which we set to be the origin. Therefore the concentration c satisfies the equation

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = \nabla^2 c, \quad (1)$$

where we have scaled x, y, so that the Peclet number is unity.

Governing Equations

And for (1), we can either

- assume inviscid, potential flow with

$$u = 2x = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}; v = -2y = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} \qquad (2)$$

  where ϕ is the velocity potential, ψ is the stream function.

- or assume viscous flow so that

$$u = xf'(y\sqrt{\text{Re}}); v = -f(y\sqrt{\text{Re}})/\sqrt{\text{Re}}, \qquad (3)$$

  where f satisfies 3^{rd} order ODE (see L.Rosenhead's "Laminar boundary layers", Page 155 - 156).

Boundary Conditions

For simplicity, we suppose smoke is injected from a source localised at $x = 0, y = -1$. Then (1) is written as

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = \nabla^2 c + \delta(x)\delta(y+1), \quad y < 0, \tag{4}$$

and one boundary condition is $c \to 0$ as $x^2 + y^2 \to \infty$. But we will need a second boundary condition at the ceiling $y = 0$.

Boundary Conditions

Here the deposition is governed by the local fluid flow, surface properties ,and especially thermophoresis (the temperature gradient drives soot particles towards colder regions, see Sheldon K. Friedlander's book "Smoke, dust, and haze : fundamentals of aerosol dynamics ").

It is traditional to 'lump' these effects into a 'deposition velocity',

$$\epsilon = \frac{1}{c}\frac{\partial c}{\partial n}, \tag{5}$$

where $\frac{\partial}{\partial n}$ is the outward derivative normal to the boundary. We expect ϵ will be small when n is measured on the room scale.

Resolutions

We can either

- take inviscid flow and transfer to ϕ, ψ as independent variable, and this gives

$$\frac{\partial c}{\partial \phi} = \frac{\partial^2 c}{\partial \phi^2} + \frac{\partial^2 c}{\partial \psi^2}, \tag{6}$$

  where

$$\psi = 2xy, \quad \phi = x^2 - y^2,$$

- or take $Re < \infty$ and solve (4) numerically using a velocity field that must be calculated numerically from the ODE mentioned above.

Asymptotic Expansion for Inviscid Problem

In (5), we assume $\epsilon \ll 1$ and set

$$c \sim c_0 + \epsilon c_1 + \cdots ,$$

where

$$\frac{\partial c_0}{\partial \phi} = \frac{\partial^2 c_0}{\partial \phi^2} + \frac{\partial^2 c_0}{\partial \psi^2} + \delta(\phi + 1)\delta(\psi). \tag{7}$$

with

$$\frac{\partial c_0}{\partial \psi} = 0 \quad \text{on} \quad \psi = 0.$$

Asymptotic Expansion for Inviscid Problem

Setting

$$c_0 = e^{(1+\phi)/2} C,$$

we find that $C = F(R) = F(\sqrt{(\phi+1)^2 + \psi^2})$, where

$$\frac{d^2 F}{dR^2} + \frac{1}{R}\frac{dF}{dR} - \frac{F}{4} = 0, \qquad (8)$$

away from the source. Then we have

$$F = \frac{1}{\pi} K_0(R/2). \qquad (9)$$

Asymptotic Expansion for Inviscid Problem

To check:

- near the source, $K_0 \sim -log(R/2) - \gamma$ as $R \to 0$, so conforming with delta function in (7).
- away from the source,

$$\left(\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial \psi^2} - \frac{\partial}{\partial \phi} \right) c_0 = 0.$$

- $y = \psi = 0$ boundary condition, from the fact

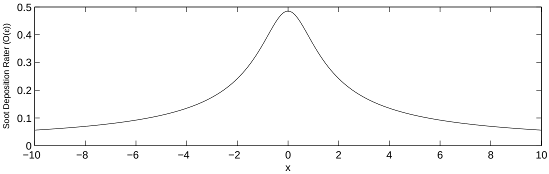
$$\frac{\partial c_0}{\partial \psi} = \frac{\partial c_0}{\partial R} \cdot \frac{\psi}{\sqrt{(\phi + 1)^2 + \psi^2}}.$$

Deposition Rate at the Roof

At the ceiling where $y = \psi = 0$, we have that the lowest order deposition rate is $\frac{\partial c_1}{\partial y}$ where, from (5)

$$\frac{\partial c_1}{\partial y} = c_0 = \frac{1}{\pi} e^{(x^2+1)/2} K_0\left(\frac{1+x^2}{2}\right) \sim O(1/x) \tag{10}$$

as $x \to \infty$ (see Fig. below).



This is the flux of soot as a function of position and hence, assuming all soot sticks to the ceiling, this will be the rate of growth of the soot layer.

Conclusion and Future Work

- Conclusion:
  This model may not be realistic in practice, but we hope it will be useful to check numerical predictions and heuristic predictions.
- Future Directions
  1. unsteady flow
  2. numerical simulations
  3. 3-D flow
  4. couple to forced convection model

Thank You