

# Properties of zeros of pseudo-ultraspherical polynomials

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The pseudo-ultraspherical polynomial of degree  $n$  can be defined by  $\mathcal{C}_n^{(\lambda)}(x) = (-i)^n C_n^{(\lambda)}(ix)$  where  $C_n^{(\lambda)}(x)$  is the ultraspherical polynomial. It is known that when  $\lambda < -n$ , the finite set  $\{C_n^{(\lambda)}(x), n = 0, 1, \dots, -\lfloor \lambda + 1 \rfloor\}$  is orthogonal on  $(-\infty, \infty)$  with respect to the weight function  $(1 + x^2)^{\lambda - \frac{1}{2}}$  and when  $\lambda < 1 - n$ , the polynomial  $\mathcal{C}_n^{(\lambda)}(x)$  has exclusively real and simple zeros. We discuss properties of the zeros of these polynomials including bounds, numbers of real zeros, monotonicity and interlacing properties.