Recent Progress and Challenging Issues on Algebraic Approximation and Approximation Theory of the p and $h-p$ Finite Element method

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In recent decades the algebraic approximation theory and the approximation theory of the h and h -p finite element methods have made substantial progresses. Among all developments two are fundamental. The first one is establishment of algebraic approximation theory in the Jacobi-weighted Besov and Sobolev spaces, and the second one is construction of the local Jacobi-weighted projection-interpolation which possesses optimality, locality, conformity and uniformity. These breakthroughes lay down the mathematical foundation of direct and inverse algebraic approximation theory and lead to the optimal convergence of the h and h -p finite element methods for problems on smooth and un-smooth domains, and answer two unresolved fundamental issues in the past many decades, i.e., what are the most appropriate function spaces for algebraic approximation and how to construct a continuous and piecewise polynomial possessing optimal error bound locally and globally for singular functions satisfying homogeneous and non-homogeneous Dirichlet boundary condition on general meshes with elements of all types.

In this talk we will present the major results on algebraic approximation theory and the approximation theory of the h and h -p finite element methods, including concepts, methodology and significance.