Mixing the Measures in Scaling Limits

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Let μ be a measure, for example, supported on $(-1,1)$. Let $\{p_n\}$ denote the associated orthogonal polynomials, and

$$K_n(x,y)=\sum_{j=0}^{n-1} p_j(x)\,p_j(y)\,,$$

be the associated nth reproducing kernel. The bulk universality limit of random matrix theory asserts that for a,b real,

$$\lim_{n\to\infty}\frac{\pi\sqrt{1-\xi^2}\mu'(\xi)}{n}K_n\left(\xi+a\frac{\pi\sqrt{1-\xi^2}}{n},\xi+b\frac{\pi\sqrt{1-\xi^2}}{n}\right)=S(a-b)\,,$$

where $S(t)=\frac{\sin\pi t}{\pi t}$. It is known true for very general measures in varying formulations. It has applications, amongst other things, to zero distribution of orthogonal polynomials and their reproducing kernels.

Suppose that ν is another measure, supported on $(-1,1)$, with orthonormal polynomials $\{q_n\}$. What can we say about scaling limits for the mixed reproducing kernel

$$K_n^{(\mu,\nu)}(x,y)=\sum_{j=0}^{n-1} p_j(x)\,q_j(y)?$$

We discuss this, and also establish a variational property for $m\times m$ determinants whose entries are mixed reproducing kernels formed from m different measures.