Blowup or no blowup? The interplay between theory and computation in the study of 3D Euler equations

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Whether the 3D incompressible Euler equations can develop a singularity in finite time from smooth initial data is one of the most challenging problems in mathematical fluid dynamics. This question is closely related to the Clay Millennium Problem on 3D Navier-Stokes Equations. We first review some recent theoretical and computational studies of the 3D Euler equations. Our study suggests that the convection term could have a nonlinear stabilizing effect for certain flow geometry. We then present strong numerical evidence that the 3D Euler equations develop finite time singularities. To resolve the nearly singular solution, we develop specially designed adaptive (moving) meshes with a maximum effective resolution of order 10^{12} in each direction. A careful local analysis also suggests that the blowing-up solution is highly anisotropic and is not of Leray type. A 1D model is proposed to study the mechanism of the finite time singularity. Very recently we prove rigorously that the 1D model develops finite time singularity. This is a joint work of Prof. Guo Luo.