Soap film dynamics and topological jumps under continuous deformation

H.K.Moffatt
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, UK
Email: hkm2@cam.ac.uk

Consider the dynamics of a soap-film bounded by a flexible wire (or wires) which can be continuously and slowly deformed. At each instant the soap-film is relaxed in quasi-static manner to a minimum-area (i.e. minimum-energy) state compatible with the boundary configuration. This can however pass through a critical configuration at which a topological jump is inevitable. We have studied an interesting example of this behaviour: the jump of a one-sided (Möbius strip) soap-film to a two-sided film as the boundary is unfolded and untwisted from the double cover of a circle. The nature of this jump will be demonstrated and explained.

More generally, dynamical systems have a natural tendency to relax through dissipative processes to a minimum-energy state, subject to any relevant constraints. An example is provided by the relaxation of a magnetic field in a perfectly conducting but viscous fluid, subject to the constraint that the magnetic field lines are frozen in the fluid. One may infer the existence of magnetostatic equilibria (and analogous steady Euler flows) of arbitrary field-line topology. In general, discontinuities (current sheets) appear during this relaxation process, and this is where reconnection of field-lines (with associated change of topology) can occur. Just as for the soap film, slow change of boundary conditions can lead to critical conditions in which such topological jumps are inevitable.

(Work in collaboration with Ray Goldstein and Adriana Pesci)