ABSTRACTS OF TALKS

Krall-type Polynomials on Non-uniform Lattices

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We will present an unified way of studying the Krall-type polynomials on non-uniform lattices, not only for the q-linear ones, but also for the q-quadratic lattice. In particular we will show a simple algorithm for getting the second order linear difference equation and the three-term recurrence relations that such polynomials satisfy. In particular the q-Racah-Krall polynomials obtained via the addition of two mass points to the weight function of the non-standard q-Racah polynomials at the ends of the interval of orthogonality will be discussed, as well as some important of their limit cases.

Some Problems on Orthogonal Polynomials where Asymptotics might be Useful

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A lot is known about the zeros and the magnitude of the classical orthogonal polynomials or Jacobi, Laguerre, and Hermite.

Much less is known about the zeros and the magnitude of the discrete versions of these and other related classical type polynomials. Some problems will be mentioned which I would like to see solved.

Turán Type Inequalities for Some Special Functions

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We extend some known higher order Turán type inequalities to Bessel functions of the first kind and we prove a Turán type inequality for the Bessel functions of the second kind. Moreover, we establish the complete monotonicity, Laguerre and Turán type inequalities for the so-called Krätzel function, which may be considered as a generalization of the reduced modified Bessel function of the second kind.

The talk is based on the papers [1] and [2].

[1] Á. Baricz, D. Jankov, T.K. Pogány, Turán type inequalities for Krätzel functions, http://arxiv.org/abs/1101.2523v1
[2] Árpád Baricz, T.K. Pogány, Turán determinants of Bessel functions, http://arxiv.org/abs/1101.4624v1

Shell Polynomials and Indeterminate Moment Problems: Answer to a Question by Ted Chihara

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The generalized Stieltjes–Wigert polynomials depending on parameters $0 \leq p < 1$ and $0 < q < 1$ are discussed. By removing the mass at zero of the N-extremal solution concentrated in the zeros of the D-function from the Nevanlinna parametrization, we obtain a discrete measure μ^M which is uniquely determined by its moments. We calculate the coefficients of the corresponding orthonormal polynomials (P_n^M). As noticed by T. Chihara, these polynomials are the shell polynomials corresponding to the maximal parameter sequence for a certain chain sequence. We also find the minimal parameter sequence, as well as the parameter sequence corresponding to the generalized Stieltjes–Wigert polynomials, and compute the value of related continued fractions. The mass points of μ^M have been studied in recent papers of Hayman, Ismail–Zhang and Huber. In the special case of $p = q$, the maximal parameter sequence is constant and the determination of μ^M and (P_n^M) gives an answer to a question posed by T. Chihara in 2001.

Counting Patterns and Asymptotic Formulas

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In many occasions counting the number of objects of finite sets parameterized by a family of integers gives rise to a polynomial function of the family of the integers. For instance in graph theory, counting the number of proper colorings of a graph by n colors gives rise to the chromatic polynomial function of n; the polynomial comes from the counting patterns n^k, the number of colorings of a k element set by n colors without restriction. If the number of elements to be counted in sets are infinite, the ordinary counting does not make sense. However, in some occasions the counting patters are still there, and give rise to power series or asymptotic formulas. In this talk, I shall exhibit a number of examples from the viewpoint of counting or measuring finite and infinite sets with structures, including an example of interpreting the generating function of partitions of integers as the total measure of the Grassmannian of infinite-dimensional subspaces of the vector space $\mathbb{K}^\infty$ over a field $\mathbb{K}$.

A q-Deformation of Nevanlinna's Value Distribution Theory

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We shall use Askey-Wilson operator to establish a new version of Nevanlinna theory for space of slow growing meromorphic functions defined in the complex plane. A new Picard-type theorem is obtained as a consequence of our theory.

Orthogonal Polynomials on Finite and Infinite Gap Sets

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The theory of orthogonal polynomials on finite unions of compact intervals can be generalized to infinite gap sets of Parreau–Widom type. This notion of regular compact sets includes Cantor sets of positive measure, among others.

When E is a Parreau–Widom set of positive measure, the equilibrium measure $d\mu_{\mathsf{E}}$ of E is absolutely continuous. By rescaling, if necessary, we may assume that the logarithmic capacity of E is 1. For probability measures $d\mu = f(t)dt + d\mu_{\mathrm{s}}$ with essential support E, we shall concentrate on the Szegő condition

$$\int_{\mathsf{E}} \log f(t)\, d\mu_{\mathsf{E}}(t) > -\infty. \tag{$*$}$$

Under certain assumptions on the mass points of $d\mu_{\mathrm{s}}$ outside E, we show that $(*)$ is equivalent to boundedness of the leading coefficients in the associated orthonormal polynomials P_n.

The question of polynomial asymptotics will also be discussed. In particular, we investigate to which extent P_n admits a power asymptotic behavior (aka Szegő asymptotics). In this connection, the isospectral torus $\mathcal{T}_{\mathsf{E}}$ of dimension equal to the number of gaps in E will be introduced as the key player. Our analysis relies on the covering space formalism introduced into spectral theory by Sodin–Yuditskii. This allows us to 'lift' functions on the multiply connected domain $\overline{\mathbb{C}} \setminus \mathsf{E}$ to the unit disk $\mathbb{D}$. The universal covering map $\mathbf{x} : \mathbb{D} \to \overline{\mathbb{C}} \setminus \mathsf{E}$ is linked with a Fuchsian group Γ of Möbius transformations in such a way that

$$\mathbf{x}(z) = \mathbf{x}(w) \iff \exists \gamma \in \Gamma :\ z = \gamma(w).$$

As we shall see, it is crucial whether or not the Abel map from $\mathcal{T}_{\mathsf{E}}$ to Γ^*, the multiplicative group of characters of Γ, is a homeomorphism.

Painleve Equations - Nonlinear Special Functions

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The Painlevé equations, discovered about a hundred years ago, are special amongst nonlinear ordinary differential equations in that they are "integrable" due to their representation as Riemann- Hilbert problems. Further they are nonlinear analogues of the classical special functions and have a plethora of remarkable properties. However, little is known about the numerical solution of the Painlevé equations due to the nonlinearity. An important question is how properties of the Painlevé equations, in particular their representation as Riemann-Hilbert problems, can be used in the development of their numerical solution.

In this talk I shall give an overview of the Painlevé equations and discuss some of their properties, in particular some recent developments. Further I shall discuss some of the "Painlevé Challenges", i.e. open problems in the field of Painlevé equations.

Linearization of Differential Systems and Matrix Valued Generalizations of Orthogonal Polynomials

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Differential systems which are holomorphic near an equilibrium point of Fuchsian type are biholomorphically equivalent, near this point, to their linear part (generically): they are linearizable.

Are systems linearizable in a larger region, containing two, or more Fuchsian points? It turns out that this is not necessarily true, but that corrections can be found after which the systems become linearizable.

In dimension one, the construction of the correction, and of the linearizing transformation, can be achieved by expansion in terms of Jacobi polynomials (in the case of two singularities), or Jacobi-Angelesco multiple orthogonal polynomials (for three or more singularities).

In higher dimensions, similar results can be obtained using new matrix valued generalizations of these polynomials, which turn out to share many properties of their scalar counterparts.

Orthogonal Polynomials and Painleve Transcendents

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Orthogonal polynomials play an important role in many branches of mathematics, including mathematical physics, approximation theory and combinatorics. Recently, it is found out that they are also related to Painleve transcendents, which are regarded as nonlinear special functions in the 21^{st} century. In this talk, I will give an introduction about their relations.

Topological Expansion in the Cubic Random Matrix Model

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We present results on the large N expansion of the free energy of a unitary random matrix model with weight $\exp(-NV(z))$, where $V(z) = -z^2/2 + uz^3$ and $u > 0$ is a real parameter. For small enough u, the free energy $F_N(u)$ can be expanded in powers of N^{-2}, the first two terms of this topological expansion are known from the work of Brézin, Itzykson, Parisi and Zuber, and they can be written in terms of hypergeometric functions. Our results are obtained by a Riemann–Hilbert analysis of the corresponding family of orthogonal polynomials in the complex plane, together with the string equations for the associated recurrence coefficients.

Summability of Jacobi Series by Uniform Lower Triangular Matrix Method

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The Jacobi polynomial $P_n^{(\alpha,\beta)}(x)$ which is obtained from Jacobi differential equation is an orthogonal polynomial over the interval $(-1, 1)$ with respect to weight function $(1-x)^\alpha(1+x)^\beta$, $\alpha > -1$, $\beta > -1$. Here Jacobi series has been taken and established a theorem on lower triangular matrix summability of a Jacobi series.

Orthogonal Polynomials and Fourier Transforms with Real Zeros

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The real polynomials with only real zeros are called hyperbolic ones. A real entire function φ is said to belong to the Laguerre-Pólya class, denoted by $\varphi \in \mathcal{LP}$, if it is a local uniform limit of a sequence of hyperbolic polynomials. We discuss questions relating Fourier transforms of certain positive kernels which belong to $\mathcal{LP}$ and properties of the corresponding polynomials, orthogonal with respect to the kernel, considered as an weight function.

Analysis of a Sequence of Orthogonal Polynomials Related to the Lommel Polynomials

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We will review some known results of the Lommel polynomials and present some properties of a family of orthogonal polynomials associated with them.

Stieltjes Interlacing of Zeros of Laguerre Polynomials from Different Sequences

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Stieltjes' Theorem proves that, given any orthogonal sequence $\{p_n\}_{n=0}^{\infty}$, if $k < n$, then between any two consecutive zeros of p_k there is at least one zero of p_n; a property called Stieltjes interlacing. We prove that Stieltjes interlacing extends across different sequences of Laguerre polynomials L_n^{α}, $\alpha > -1$. In particular, we show that Stieltjes interlacing holds between the zeros of $L_{n-1}^{\alpha+t}$ and L_{n+1}^{α}, $\alpha > -1$, when $t \in \{1, \ldots, 4\}$ but not in general when $t > 4$ and provide numerical examples to illustrate the breakdown of interlacing. More generally, we show Stieltjes interlacing occurs between the zeros of L_{n+1}^{α} and the zeros of the kth derivative of L_n^{α}, as well as with the zeros of $L_{n-k}^{\alpha+k+t}$ for $t \in \{1, 2\}$. In each case, we identify associated polynomials, analogous to the de Boor-Saff polynomials, that are completely determined by the coefficients in a mixed three term recurrence relation and whose zeros complete the interlacing process.

Rodrigues Formula for Orthogonal Matrix Polynomials Satisfying Differential Equations

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The theory of matrix valued orthogonal polynomials was started by M. G. Krein in 1949. But more than 50 years have been necessary to see the first examples of orthogonal matrix polynomials $(P_n)_n$ satisfying second order differential equations of the form

$$P_n''(t)F_2(t) + P_n'(t)F_1(t) + P_n(t)F_0 = \Gamma_n P_n(t). \tag{1}$$

Here F_2, F_1 and F_0 are matrix polynomials (which do not depend on n) of degrees less than or equal to 2, 1 and 0, respectively. These families of orthogonal matrix polynomials are among those that are likely to play in the case of matrix orthogonality the role of the classical families of Hermite, Laguerre and Jacobi in the case of scalar orthogonality.

This talk is devoted to the question of the existence of Rodrigues' formulas for these families of orthogonal matrix polynomials, that is, assuming that the sequence of orthogonal matrix polynomials $(P_n)_n$ with respect to W satisfies the set of differential equations (1), $n \geq 0$, is there any efficient and canonical way to produce the sequence of polynomials $(P_n)_n$ from W and the differential coefficients F_2, F_1 and F_0? (Say in an analogous way as to the formula

$$p_n = (f_2^n w)^{(n)}/w,$$

produces the orthogonal polynomials with respect to a classical scalar weight w).

Matrix Valued Szegő Polynomials and Quantum Walks

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The use of these polynomials allows one to give a version of the well known Karlin-McGregor formula for classical random walks.

Using this one can study recurrence and localization properties of quantum walks, a subject that has attracted interest in physics as well as computer science. These models are starting to be used in different fields.

This is joint work with MJ Cantero, L Moral and L Velazquez.

Generalized Stirling Numbers and Generalized Stirling Functions

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Here presented is a unified approach to Stirling numbers and their generalizations as well as generalized Stirling functions by using k-Gamma functions. Previous well-known extensions of Stirling numbers due to Riordan, Carlitz, Howard, Charalambides-Koutras, Gould-Hopper, Hsu-Shiue, Tsylova Todorov, Ahuja-Enneking, and the Stirling functions introduced by Butzer and Hauss, Butzer, Kilbas, and Trujilloet and others are included as particular cases of our generalization. Some basic properties related to our general pattern such as their recursive relations, generating functions, and asymptotic behavior are discussed. Two algorithms for calculating the Stirling numbers based on our generalization are also given.

Szego's Theorem for Matrix Orthogonal Polynomials

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Matrix orthogonal polynomials arise in a variety of settings, most notably in connection with multivariate random processes. The analytic theory of such polynomials on the unit circle has emerged quite recently and is developing fast. I will discuss some new results that generalize the classical formulas of Gabor Szego and others to the matrix setting. This is joint work with Maxim Derevyagin, Sergey Khrushchev and Mikhail Tyaglov.

Convergence Acceleration Algorithms via Discrete Integrable Systems

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As is known, extrapolation methods are used to accelerate the convergence of a given sequence of numbers, vectors or matrices. They play an important role in many branches of numerical analysis, such as approximation of functions, numerical integration, or discretization methods for ODEs. The applicability of extrapolation methods is connected with the existence of an asymptotic expansion of the sequence under consideration. On the other hand, the theory of integrable systems has been an active area of mathematics for the past fourty years. Different aspects of the subject have fundamental relations with mechanics, applied mathematics, algebraic structures, theoretical physics, analysis, geometry and so on. The study of the connections and their interplays between extrapolation method and integrable systems will be very profitable to the fields of integrable systems and numerical analysis. In this talk, we will show some recent results on convergence acceleration algorithms via discrete integrable systems.

Multivariable H-Function

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The object of this paper is to derive formulas for the fractional integration of multivariable H-function. We shown general Eulerian integral formulas for various families of generalized hypergeometric functions of multivariables. Some of these applications of the key formulas would provide potentially useful generalizations of known results in the theory of fractional calculus.

Stieltjes, Poisson and Other Integral Representations for Functions of Lambert W

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We show that many functions containing W are Stieltjes functions. Explicit Stieltjes integrals are given for functions $1/W(z)$, $W(z)/z$, and others. We also prove a generalization of a conjecture of Jackson, Procacci & Sokal. Integral representations of W and related functions are also given which are associated with the properties of their being Pick or Bernstein functions. Representations based on Poisson and Burniston–Siewert integrals are given as well.

Integral Representations of Hypergeometric Functions

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The Euler integral representation of the ${}_2F_1$ Gauss hypergeometric function is well known and plays a prominent role in the derivation of transformation identities and in the evaluation of ${}_2F_1(a, b; c; 1)$ among other applications. Other integral representations involving the ${}_2F_1$ function include Barnes' integral representation and several integrals due to Erdélyi. The general ${}_{p+k}F_{q+k}$ hypergeometric function has an integral representation where the integrand involves ${}_pF_q$. We discuss several integral representations for some ${}_pF_q$ hypergeometric functions and examine some special cases which correspond to various transformations involving the Riemann zeta function and beta function.

Discrete and Continuous Painlevé Equations

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The classical Painleve equations are now widely used as non-linear special functions. The discrete Painleve equations are less widely known even though they possess almost all of the same properties that make the classical ODEs special. I will survey what we know about the continuous and discrete Painleve equations, touch on some new results and point out some major open questions.

New Characterizations of Jacobi, Generalized Chebyshev and Ultraspherical Polynomials

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Let $(R_n(x))_{n\in\mathbb{N}_0}$ be an orthogonal polynomial sequence WRT a probability measure μ on $\mathbb{R}$, where we assume that $|\text{supp }\mu| = \infty$, $\text{supp }\mu \subseteq [-1,1]$ and $R_n(1) = 1$ ($n \in \mathbb{N}_0$). A theorem of R. Lasser and J. Obermaier (2008) yields that, provided μ is symmetric, $(R_n(x))_{n\in\mathbb{N}_0}$ belongs to the class of ultraspherical polynomials if and only if both equations

$$\int_{\mathbb{R}} R'_{2n-1}(x)[R_{2n-2}(x) - R_{2k-2}(x)]\, d\mu(x) \overset{(1)}{=} 0 \overset{(2)}{=} \int_{\mathbb{R}} R'_{2n}(x)[R_{2n-1}(x) - R_{2k-1}(x)]\, d\mu(x)$$

are satisfied for all $k,n \in \mathbb{N}$, $k \leq n$. Recently, discrete and continuous q-ultraspherical polynomials have been studied by M. E. H. Ismail and Obermaier in a related way. In this talk, we first present a new and shorter proof for Lasser's and Obermaier's result, which requires deeper theory of classical orthogonal polynomials, but includes an extension to the non-symmetric case of Jacobi polynomials. We then develop more sophisticated arguments and sharpen the theorem. More precisely, we prove that under the assumption of symmetry the ultraspherical polynomials are completely described by (1), while (2) characterizes the larger class of generalized Chebyshev polynomials. It is worth transferring a slight improvement of the latter result to the Jacobi polynomials again, which shall be achieved by the usage of kernel polynomials and quadratic transformations. In the last part of the talk, we apply these crucial tools, too, but shift our focus from the derivative $\frac{d}{dx}$ to the operator $(1-x^2)\frac{d}{dx}$. This approach reflects the well-known backward shift operators for Jacobi resp. ultraspherical polynomials, and it will enable us to deduce characterization theorems which have the advantage that the special normalization of $(R_n(x))_{n\in\mathbb{N}_0}$ chosen above is naturally overcome.

Log-convexity and Log-concavity for Series in Products and Ratios of Rising Factorials and Gamma Functions

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Consider the following general problems: under what conditions on the positive sequence $\{f_k\}_{k=0}^{\infty}$ and non-negative numbers $a_1, \ldots, a_n$, $b_1, \ldots, b_m$ the functions:

$$\mu \to \sum_{k=0}^{\infty} f_k \frac{(a_1+\mu)_k \cdots (a_n+\mu)_k}{(b_1+\mu)_k \cdots (b_m+\mu)_k} \tag{1}$$

and

$$\mu \to \sum_{k=0}^{\infty} f_k \frac{\Gamma(a_1+\mu+k) \cdots \Gamma(a_n+\mu+k)}{\Gamma(b_1+\mu+k) \cdots \Gamma(b_m+\mu+k)} \tag{2}$$

are log-concave or log-convex? Here Γ is Euler's gamma function and $(a)_k = a(a+1)\cdots(a+k-1) = \Gamma(a+k)/\Gamma(a)$ is rising factorial. We give complete solution to this problem for $n+m=1$ and nearly complete solution for $n=m=1$. In fact we prove more: setting $f_k = \alpha_k x^k$ we show that certain differences of products of the functions (1), (2) are absolutely continuous in x. This leads to virtually unimprovable sufficient conditions for log-convexity and log-concavity of generalized hypergeometric functions as functions of parameters. Many known and new inequalities for Bessel, Kummer and Gauss functions are included in these results as particular cases. If we allow for negative values of parameters $a_1, \ldots, a_n$, $b_1, \ldots, b_m$ and for the minus sign in front of some μs in (1), (2) we arrive at a problem which includes the classical Turán inequalities for orthogonal polynomials. However, the methods developed in the talk do not work in this situation. Further, we discuss a number of conjectures for coefficients and zeros of polynomials and rational functions intimately related to the differences of products of the functions (1), (2).

The work has been supported by the Russian Basic Research Fund (grant 11-01-00038-a) and Far Eastern Branch of the Russian Academy of Sciences (grants 09-III-A-01-008 and 11-III-B-01M-004)

Mock Theta Functions via Ramanujan's Reciprocity Theorem

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In earlier work, we showed that two mock Jacobi forms add to a Jacobi theta function using Ramanujan's three variable reciprocity theorem.

In this talk, we revisit the reciprocity theorems to study its further relation with mock theta functions.

Generalized Stieltjes Transform and Hypergeometric Functions

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Generalized Stieltjes transform of order $\alpha > 0$ of a non-negative measure $d\mu$ supported on $[0,\infty)$ is defined by

$$f(z) = \int\limits_{[0,\infty)} \frac{d\mu(u)}{(u+z)^{\alpha}} + \mu_{\infty}, \tag{1}$$

where it is assumed that $\int_{[0,\infty)} d\mu(u)/(1+u)^{\alpha} < \infty$. In the talk we discuss various properties of the class S_{α} comprising functions representable by (1) and the class $S = \bigcup_{\alpha>0} S_{\alpha}$. We demonstrate the connection between different definitions occurring in the literature and relations between representations of the same function by the integral (1) with differing values of α. We also introduce the notion of exact Stieltjes order by

$$\alpha^{*}(f) = \inf\{\alpha : \ f \in S_{\alpha}\}$$

and provide a simple criterion of exactness. In the second part of the talk we present a generalized Stieltjes transform representation of the generalized hypergeometric function ${}_{q+1}F_q((a_{q+1});(b_q);z)$ and give some results on its exact Stieltjes order. Moreover, we discuss some mapping properties of ${}_{q+1}F_q((a_{q+1});(b_q);z)$ and their relation to $\alpha^{*}({}_{q+1}F_q)$.

Study of Two-Variable Forms of Hermite, Laguerre and Legendre Polynomials using Lie-Mon Method

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Orthogonal polynomials include many special functions commonly encountered in the applications, for example, Hermite, Laguerre and Legendre polynomials. These polynomials have extremely diverse applications in physics and engineering. The Lie-algebraic methods and the monomiality formalism provide a powerful tool to study the properties of special functions, in particular of special polynomials. This combination, referred from now on as Lie-Mon, has been developed by Dattoli and Khan to treat families of classical and non-classical polynomials within a general and unifying framework. This method is extended to deal with two-variable forms of Hermite, Laguerre and Legendre polynomials, which opens new possibilities to deal with their theoretical foundations. An account of the technicalities associated with Lie-Mon method is presented and then applied to get new forms of generating functions for the Hermite, Laguerre and Legendre family.

Spectral Properties of Differential Operators using Orthogonal Polynomials

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The classical theorem by Bochner gives the second order differential operators having polynomial eigenfunctions. Generalising Bochner's approach we look at differential operators for which there exists a suitable basis of functions tridiagonalising the differential operator.

This gives the opportunity to describe the spectrum of the differential operators involved. We illustrate the approach by several examples, and we discuss generalisations to other types of operators. This is joint work with Mourad Ismail.

Differentiation by Integration using Orthogonal Polynomials

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This survey lecture (joint work with Enno Diekema, see [2]) discusses the history of approximation formulas for n-th order derivatives by integrals involving orthogonal polynomials. There is a large but rather disconnected corpus of literature on such formulas. The pioneers, who worked independently of each other, are Cioranescu (1938), Haslam-Jones (1953), Lanczos (1956) and Savitzky & Golay (1964).

Applications which motivated these formulas are as diverse as numerical mathematics, spectroscopy, electrical engineering, filter theory and actuarial science. I will somewhat generalize the various results in literature, in particular unifying the continuous and discrete case. Many side remarks will be made, for instance on wavelets, Mantica's Fourier-Bessel functions, Greville's minimum R_α formulas in connection with discrete smoothing, and the incredible history of an identity incorrectly ascribed in [1] to Chaundy & Bullard.

[1] T.H. Koornwinder and M.J. Schlosser, On an identity by Chaundy and Bullard. I, Indag. Math. (N.S.) 19 (2008), 239-261; arXiv:0712.2125v3.

[2] E. Diekema and T.H. Koornwinder, Differentiation by integration using orthogonal polynomials, a survey, arXiv:1102.5219v1.

Software for Special Functions

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Many special functions are solutions of linear differential equations and recurrences. Therefore they belong to the class of holonomic functions for which many computer algebra algorithms exist. We present recently developed software for dealing with special functions and show some applications thereof in different areas of mathematics. Our software HolonomicFunctions assists in proving identities which involve integrals and symbolic sums, and is able to compute relations of various forms for special functions. It also contributed to solve some notorious problems from combinatorics.

Orthogonal Polynomials in the Normal Matrix Model

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The normal matrix model is a random matrix model defined on complex matrices. The eigenvalues in this model fill a two-dimensional region in the complex plane as the size of the matrices tends to infinity. Orthogonal polynomials with respect to a planar measure are a main tool in the analysis.

In many interesting cases, however, the orthogonality is not well-defined, since the integrals that define the orthogonality are divergent. I will present a way to redefine the orthogonality in terms of a well-defined Hermitian form. This reformulation allows for a Riemann-Hilbert characterization. For the special case of a cubic potential it is possible to do a complete steepest descent analysis on the Riemann-Hilbert problem, which leads to strong asymptotics of the orthogonal polynomials, and in particular to the two-dimensional domain where the eigenvalues are supposed to accumulate.

This is joint work with Pavel Bleher (Indianapolis).

Convolution Structures Generated by Orthogonal Polynomial Sequences: Application to Stochastic Sequences

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The linearization coefficients of products of two orthogonal polynomials often are the fundament for the definition of convolution structures on the natural numbers. In a first part we present results which refer to this convolution and lead to polynomial hypergroups or signed polynomial hypergroups or semibounded polynomial hypergroups, respectively. In addition we give a lot of examples of orthogonal polynomial systems enjoying these properties. In a second part we study stochastic sequences which fulfill stationarity conditions determined by the linearization coefficients. We present an ergodic theorem and a multiplier theorem for such stochastic sequences.

Uniform Asymptotic Expansions for Second-order Linear Difference Equations with Turning Points

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Two linearly independent asymptotic solutions are constructed for the second-order linear difference equation

$$y_{n+1}(x) - (A_n x + B_n)y_n(x) + y_{n-1}(x) = 0,$$

where A_n and B_n have power series expansions of the form

$$A_n \sim n^{-\theta} \sum_{s=0}^{\infty} \frac{\alpha_s}{n^s}, \qquad B_n \sim \sum_{s=0}^{\infty} \frac{\beta_s}{n^s}$$

with $\theta \neq 0$ being a real number and $\alpha_0 \neq 0$. Upon a rescaling $x = n^\theta t$, two turning points $t_\pm$ are defined by $\alpha_0 t_\pm + \beta_0 = \pm 2$. In particular, it is shown that how the Bessel functions J_ν and Y_ν arise in the uniform asymptotic expansions around the turning point $t_- = 0$. As an illustration of the main result, a uniform asymptotic expansion is derived for the orthogonal polynomials associated with the Laguerre type weights $w(x) = x^\alpha \exp(-Q(x))$, where $\alpha > -1$ and Q denotes a polynomial with positive leading coefficient.

A Generating Function for the Askey-Wilson Polynomials

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We give a new generating function for the Askey-Wilson polynomials, which allows us to give a new proof of the orthogonality of the Askey-Wilson polynomials.

On a Nonorthogonal Polynomial Sequence Generated by Bessel Operator

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A polynomial sequence generated by powers of the Bessel operator will be under discussion. The integral representation along with the generating function will be revealed via the Kontorovich-Lebedev transform. After presenting an explicit expression for such polynomial sequence, the corresponding dual sequence will be thoroughly constructed. It turns out that the canonical element of this dual sequence is a linear functional associated to a positive-definite measure. Finally, we build the corresponding orthogonal polynomial sequence.

(Joint work with P. Maroni and S. Yakubovich).

Monotonicity of Zeros of Laguerre-Sobolev Type Orthogonal Polynomials

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In this paper we analyze the location of zeros of polynomials orthogonal with respect to the inner product

$$\langle p, q\rangle = \int_0^\infty p(x)q(x)x^\alpha e^{-x}dx + Np^{(j)}(a)q^{(j)}(a), \tag{1}$$

where $\alpha > -1$, $N \geq 0$, a is a negative real number, and $j \in \mathbb{N}$.

In particular, when $a = 0$, we focus our attention in their interlacing properties with respect to the zeros of Laguerre polynomials as well as in the monotonicity of each individual zero in terms of the mass N. We give necessary and sufficient conditions in terms of N in order the least zero of any Laguerre-Sobolev type orthogonal polynomial be negative.

Finally, taking into account numerical experiments, some open problems and conjectures about the behavior of the zeros of Laguerre- Sobolev type orthogonal polynomials when $a < 0$ are stated.

This is a joint work with E. Huertas (Universidad Carlos III de Madrid) and F. Rafaelli (UNESP, Brazil)

Duplication Formulae for Jacobi Theta Functions via Gosper's q-Trigonometry

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In 2001 R. W. Gosper introduced a kind of q-trigonometric functions. These are connected to the Jacobi thetas as well. Gosper deduced and conjectured a number of identities involving his functions. In addition, he phrased a number of questions (which he called "mysteries") about the new q-sine and q-cosine function.

We answer his questions in the affirmative and thus we prove new duplication formulae for the old functions of Jacobi. These investigations enable us to present some interesting infinite sums involving the Jacobi thetas.

Moreover, we present a construction of new q-exponential and q-hyperbolic functions, hence we reveal the solutions to Gosper's "mysteries".

Appell-Lerch Sums, Hecke-type Double Sums, and Mock Theta Functions

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Here we prove a formula which expands a family of Hecke-type double sums in terms of Appell-Lerch sums and theta functions. This formula reduces many classical Hecke-type double sum and mock theta function identities to straight-forward exercises. We also show how this formula builds on recent results of Andrews on q-orthogonal polynomials, Rogers-Ramanujan identities, and mock theta functions. This is joint work with Dean Hickerson.

Nicholson-type Formulas for Special Functions

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Nicholson's formula for Bessel functions

$$J_\nu^2(z) + Y_\nu^2(z) = \frac{8}{\pi^2} \int_0^\infty K_0(2z \sinh t) \cosh 2\nu t \, dt, \quad \mathrm{Re}\, z > 0,$$

and some similar formulas are useful in providing information about zeros of these functions. In the 1930s, A. L. Dixon, W. L. Ferrar and T. W. Chaundy investigated methods of discovering and proving such relations based on factorization of differential operators. I will report on further investigations of this kind including formulas for products of special confluent hypergeometric functions.

Chebyshev-Blaschke Products

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In a recent joint work with Mingxi Wang, a version of Ritt's theory on the factorization of finite Blaschke products has been developed. In this Ritt's theory on the unit disk, a special class of finite Blaschke products has been introduced as the counterpart of Chebyshev polynomials in Ritt's theory for polynomials. These special finite Blaschke products are therefore called Chebyshev-Blaschke products. In this talk, I will explain the construction of them and also discuss some of their interesting properties.

Multidimensionally Consistent Systems and Painleve and Garnier Transcendents

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I will review some insights obtained over the last few years on multidimensionally consistent integrable discrete and continuous systems of nonlinear equations and their reductions to discrete or continuous Painleve and Garnier type systems. In particular the Lagrangian aspects of multidimensionally consistent systems will be surveyed, and the problem of how the latter reduce to the relevant Lagrange structures for the corresponding Painleve and Garnier systems, will be addressed.

Convergence and Absolute Convergence of Fourier Series with Respect to Orthogonal Polynomials

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In the proceedings of the International Workshop 'Special Functions' hold June 21 - 25th, 1999, at the City University of Hong Kong there is a proof that, concerning little q-Legendre polynomials, the set of functions with absolute convergent Fourier series is a proper subset of the set of continuous functions [1]. It is straightforward to check that this is even true for any orthogonal polynomial system with respect to a measure with compact support. Moreover, studying Fourier series in the context of so-called harmonic Banach spaces one is able to show that the set of functions with absolute convergent Fourier series is a proper subset of the set of functions with convergent Fourier series in any case.

References

[1] R. Lasser, J. Obermaier, Orthogonal expansions for L^p and C-spaces, in: Special Functions, Proceedings of the International Workshop, World Scientific Publishing Singapore (2000) 194-206.

Two-parameter Deformations of Multivariate Hook Product Formulae

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Let $\lambda = (\lambda_1, \cdots, \lambda_r)$ be a partition. A reverse plane partition of shape λ is an array of non-negative integers

$$\begin{array}{cccccc} \pi_{1,1} & \pi_{1,2} & & \cdots\cdots & & \pi_{1,\lambda_1} \\ \pi_{2,1} & \pi_{2,2} & & \cdots & \pi_{2,\lambda_2} & \\ \vdots & \vdots & & & & \\ \pi_{r,1} & \pi_{r,2} & \cdots & \pi_{r,\lambda_r} & & \end{array}$$

with weakly increasing rows and columns. E. Gansner used the Hillman–Grassl correspondence to prove the following hook product formula for the multivariate (trace) generating function of reverse plane partitions:

$$\sum_{\pi} \boldsymbol{z}^{\mathrm{tr}(\pi)} = \prod_{v \in D(\lambda)} \frac{1}{1 - \boldsymbol{z}[H_{D(\lambda)}(v)]},$$

where π runs over all reverse plane partitions of shape λ and $\boldsymbol{z}^{\mathrm{tr}(\pi)} = \prod_{i,j} z_{j-i}^{\pi_{ij}}$. And $\boldsymbol{z}[H_{D(\lambda)}(v)] = \prod_{(i,j) \in H_{D(\lambda)}(v)} z_{j-i}$ is the monomial associated to the hook $H_{D(\lambda)}(v)$ of v in the diagram $D(\lambda)$ of λ.

In this talk, I will give another proof to Gansner's formulae for shapes and shifted shapes by using operator calculus on the ring of symmetric functions. This proof leads to a (q,t) deformation of Gansner's formulae of the form

$$\sum_{\pi} W(\pi; q, t) \boldsymbol{z}^{\pi} = \prod_{v} \frac{(t\boldsymbol{z}[H(v)]; q)_\infty}{(\boldsymbol{z}[H(v)]; q)_\infty},$$

where the weight $W(\pi; q, t)$ is a product of factors of the form $\left(\dfrac{1 - q^k t^{l+1}}{1 - q^{k+1} t^l} \right)^{\pm 1}$ and it comes from the theory of Macdonald functions.

Also I will present a conjectural deformation of Peterson-Proctor's hook product formula for P-partitions on d-complete posets.

Uniform Asymptotic Approximations for the Meixner-Sobolev Polynomials

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We obtain uniform asymptotic approximations for the monic Meixner-Sobolov polynomials $S_n(x)$. These approximations for $n \to \infty$, are uniformly valid for x/n restricted to certain intervals, and are in terms of Airy functions. We also give asymptotic approximations for the location of the zeros of $S_n(x)$, especially the small and the large zeros are discussed. As a limit case we also give a new asymptotic approximation for the large zeros of the classical Meixner polynomials.

The method is based on an integral representation in which a hypergeometric functions appears in the integrand. After a transformation the hypergeometric can be uniformly approximated by unity, and all that remains are simple integrals for which standard asymptotic methods are used.

On the Uniform Asymptotic Expansions for Discrete Chebyshev Polynomials

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The discrete Chebyshev polynomials $t_n^N(x)$, which is also the special case of Hahn polynomial $Q_n(x; \alpha, \beta, N)$ with $\alpha = \beta = 0$, have a double integral representation. An asymptotic expansion is derived from this double integral representation. This expansion is given in terms of confluent hypergeometric functions when $x > 0$ and Gamma functions when $x < 0$, for all fixed $n/N \in (0, 1)$.

An Algorithmic Approach to Special Functions Inequalities

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Special function inequalities arise in many different areas of mathematics and physics. Proving them is usually non-trivial and requires in many cases in addition to ad hoc ideas, a profound knowledge of relations and transformations of hypergeometric functions. Symbolic methods for proving special function inequalities are rare or nonexistent. The only convincing procedure using computer algebra to this day, is the recent approach introduced by Gerhold and Kauers which uses Cylindrical Algebraic Decomposition (CAD). This method proves inequalities on sequences that are defined by a recurrence and initial values.

In this talk we present the basic ideas of this approach and give a non-trivial example on which the method succeeded and for which to this day there is no computer-free proof.

The procedure is not yet an algorithm in the sense that in the general case no criteria for termination are known. We will comment on this fact and present some recent results obtained together with Manuel Kauers on cases where the method is known to terminate. This analysis is based on the asymptotic behaviour of the given sequence.

Complete Monotonicity of Functions Involving Divided Differences of Polygamma Functions

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In this talk, we survey complete monotonicity of functions involving divided differences of polygamma functions, including retrospecting the origin and motivation and summarizing newly-developed works and applications.

Semiclassical Biorthogonal Elliptic Functions

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One of the reasons why the Painlevé transcendents have appeared in much recent work is their connection to semiclassical orthogonal polynomials (norms, recurrence coefficients, coefficients of differential equations, etc.). Just as the classical orthogonal polynomials can be extended to a large family of hypergeometric orthogonal polynomials (the q-Askey scheme), so can the semiclassical theory. At the top of the classical hierarchy are the elliptic biorthogonal functions; I'll discuss how to generalize the semiclassical theory to this case (and thus, taking limits, to any lower-level case), and the implications of this construction for the elliptic Painlevé equation and generalizations thereof.

Gevrey and Exact Asymptotic Expansions, Stokes Phenomena, Resurgence and Discrete Analog.

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We shall present Gevrey and exact asymptotic expansions with some applications, linear and non linear Stokes-phenomena and we will give an idea of the relation between Stokes phenomena and resurgence. Afterwards we shall present some recent works on the q-analog (q-Gevrey and exact asymptotic expansions, q-Stokes phenomena, q-alien derivations...) based upon a notion of q-asymptotics, in relation with q-special functions.

Orthogonal Laurent Polynomials and Szegő Polynomials Associated with q-Hypergeometric Functions: Orthogonality and Asymptotics

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A three-complex-parameter class of orthogonal Laurent polynomials on the unit circle associated with q-hypergeometric or basic hypergeometric functions is considered. The starting point for our analysis is the three term contiguous relation

$$\begin{aligned} {}_2\Phi_1(q^a, q^{b+1}; q^c; q, z) &= \big(1 + \frac{1-q^{a-b}}{1-q^c}\, q^b z\big)\ {}_2\Phi_1(q^{a+1}, q^{b+1}; q^{c+1}; q, z) \\ &\quad - \frac{(1-q^{a+1})\,(1-q^{c-b})}{(1-q^c)\,(1-q^{c+1})}\, q^b\, z\ {}_2\Phi_1(q^{a+2}, q^{b+1}; q^{c+2}; q, z). \end{aligned}$$

which holds for $c \neq 0, -1, -2, \ldots$.

To be precise, we obtain the L-orthogonal and asymptotic properties of the sequence of q-hypergeometric polynomials $\{\,{}_2\Phi_1(q^{-n}, q^{b+1};\, q^{-c+b-n};\, q, q^{-c+d-1}z)\}_{n=0}^{\infty}$, where $0 < q < 1$ and the complex parameters b, c and d are such that $b \neq -1, -2, \ldots$, $c - b + 1 \neq -1, -2, \ldots$ and $\mathcal{R}e(c+2) > \mathcal{R}e(d) > 0$. Explicit expressions for the recurrence coefficients, moments, orthogonality and also asymptotics are given.

With the restriction $c + 2 = d + \bar{d} = b + \bar{b} + 1 > 0$ on the parameters, information regarding a class of Szegő polynomials are also derived.

Heckman-Opdam Polynomials and the Segal-Bargmann Transform

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Heckman-Opdam polynomials are multivariable generalizations of the classical Jacobi polynomials. They are polynomial eigenfunctions of differential-reflection operators associated with root systems (the Cherednik operators), and generalize the spherical functions of Riemannian symmetric spaces of compact type. In this talk, we report on some recent results in harmonic analysis associated with Heckman-Opdam polynomials. The focus will be on a version of the Segal-Bargmann transform in this setting.

The talk is based on joint work with H. Remling.

References:

H. Remling, M. Rösler: The Heat Semigroup in the Compact Heckman-Opdam Setting and the Segal-Bargmann Transform. IMRN advance access, doi:10.1093/imrn/rnq239.

Modular-invariant Eigenfunctions of the Relativistic Calogero-Moser Hamiltonians

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The quantum relativistic Calogero-Moser N-particle system is characterized by N commuting analytic difference operators. In the hyperbolic and elliptic regimes, these difference operators depend on two scale parameters a_+ and a_-, and the interchange of a_+ and a_- yields a family of N difference operators commuting with the previous family. The seminar is concerned with joint eigenfunctions of the two families. The eigenfunctions are defined in terms of the hyperbolic and elliptic gamma functions, for which the (modular) invariance under interchange of a_+ and a_- is built in. For the hyperbolic N-particle case with $N > 2$, we report ongoing work with M. Hallnäs.

Infinite Families of Exceptional Orthogonal Polynomials

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In recent two years, various infinite families of new orthogonal polynomials are discovered [1, 2] as the main part of the eigenfunctions of *exactly solvable* Schrödinger equations and their difference analogues [3]. These new polynomials $\{P_{\ell,n}(\eta)\}$, $\ell = 1, 2, \ldots$, $n = 0, 1, 2, \ldots$, satisfy second order differential or difference equations without contradicting Bochner's theorem, since they start at degree $\ell \geq 1$. $P_{\ell,n}(\eta)$ is a degree $\ell + n$ polynomial in η.

The guiding principle is exactly solvable quantum mechanics and its discrete analogues. The exactly solvable Hamiltonian systems corresponding to the classical polynomials, the Lagurre, Jacobi, Wilson, Askey-Wilson, Racah and q-Racah polynomials, are deformed in terms of a degree ℓ eigenpolynomials with *twisted* parameters. The parameters are so chosen that the deformed system retain the shape invariance, too. The new deformed (X_ℓ) Jacobi polynomials are *global solutions* [4] of a Fuchsian differential equation with $3 + \ell$ regular singularities, in which the extra ℓ singularities are the zeros of the deforming polynomial. It should be stressed that global solutions of a Fuchsian differential equation with more than four regular singularities had been utterly unknown. The confluent limit of the X_ℓ Jacobi polynomials are the X_ℓ Laguerre polynomials. The Darboux-Crum transformations connecting the classical orthogonal polynomials with the new X_ℓ polynomials are also discovered [5, 2]. The lowest ($\ell = 1$) members of the X_ℓ Laguerre and Jacobi polynomials correspond to those introduced in [6]. This talk is based on collaboration with S. Odake, C-L Ho, S. Tsujimoto and A. Zhedanov.

References

[1] S. Odake and R. Sasaki, *Phys. Lett.* **B679** (2009) 414-417, *Phys. Lett.* **B684** (2009) 173-176, *Phys. Lett.* **B682** (2009) 130-136.

[2] S. Odake and R. Sasaki, *Prog. Theor. Phys.* **125** (2011) May issue.

[3] R. Sasaki, *Phil. Trans. R. Soc.* **A369** (2011) 1301-1318.

[4] C-L. Ho, S. Odake, & R. Sasaki, "Properties of the exceptional (X_ℓ) Laguerre and Jacobi polynomials," `arXiv:0912.5447`.

[5] R. Sasaki, S. Tsujimoto and A. Zhedanov, *J. Phys.* **A43** (2010) 315204 (20 pp), *J. Phys.* **A43** (2010) 335201 (18 pp).

[6] D. Gómez-Ullate, N. Kamran and R. Milson, *J. Approx Theory* **162** (2010) 987-1006; *J. Math. Anal. Appl.* **359** (2009) 352-367; C. Quesne, *J. Phys.* **A41** (2008) 392001; B. Bagchi, C. Quesne and R. Roychoudhury, *Pramana J. Phys.* **73** (2009) 337-347.

Quantum B-Splines

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We develop the theory of quantum or q-splines. Quantum splines are piecewise polynomials whose q-derivatives up to some order agree at the knots. We introduce the q-de Boor algorithm for q-B-spline construction using q-blossoming and homogenization. We show that every q-spline is a q-B-spline. We study some basic properties of these splines including interpolation and approximation properties, and knot insertion algorithms.

Moments of the Askey-Wilson Polynomials and the ASEP

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The moments of the Askey-Wilson polynomials are rational functions in 5 parameters, a, b, c, d and q. The asymmetrical exclusion process (ASEP) is a finite Markov chain also depending on 5 parameters. An algebraic transformation connects the steady state probabilities of the ASEP to the Askey-Wilson moments. A combinatorial model for the steady state probabilities is given in terms of tableaux.

Models of Finite Oscillators: the Hahn Oscillators

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New models for the finite one-dimensional harmonic oscillator are proposed based upon two deformations of the Lie algebra $\mathfrak{u}(2)$ extended by a parity operator, with deformation parameter α. Classes of irreducible unitary representations of $\mathfrak{u}(2)_\alpha$ are constructed. In the finite oscillator models, the eigenvalues (the (discrete) spectrum) and the eigenstates of the position operator are computed in explicit form. The components of the position eigenvectors turn out to be Hahn polynomials with parameters $(\alpha, \alpha + 1)$ and $(\alpha + 1, \alpha)$. The construction of the position eigenstates relies on two new difference equations for Hahn polynomials. The explicit knowledge of the position eigenstates and of the oscillator Hamiltonian eigenstates leads to the computation of the position wavefunctions which turn out to be dual Hahn polynomials. Plots of these discrete wavefunctions display interesting properties, similar to those of the parabose oscillator. We show indeed that in the limit, when the dimension of the representations goes to infinity, the discrete wavefunctions tend to the continuous wavefunctions of the parabose oscillator.

A Selection of Problems from Asymptotic Analysis of Integrals

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We discuss the following topics.

1. Problems with special attention to cases where we need certain special functions for obtaining the main approximants. This happens when neighboring critical points of the integrand, such as poles and other singularities, are located near an endpoint, near a saddle point, and so on.

2. We indicate how Van der Corput's approach using neutralizers in the stationary phase method can be avoided by going into the complex plane.

3. Two elliptic 3D singular perturbation problems that can be solved in terms of integrals, one in the form of a double integral with saddle points close to a line of poles in the 2D complex plane. Some aspects of Laplace's method for double integrals will be discussed.

4. The large parameter and variable behavior of certain hypergeometric functions that arise in the modeling of large ecological communities

5. The large degree behavior of generalized Bessel polynomials.

Finite Analogues of Rogers-Ramanujan Type Identities

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The first Rogers–Ramanujan identity can be stated as follows: The partitions of an integer n in which the difference between any two parts is at least 2 are equinumerous with the partitions of n into parts congruent to 1 or 4 modulo 5.

Gordon generalized this result from modulo 5 to one involving arbitrary odd k and an analytic form was found by Andrews. Since the pioneering work of Andrews, many infinite families of identities of similar type have been found.

In this talk, we will consider some polynomial analogues of these Rogers- Ramanujan type identities.

(Joint work with S. Ole Warnaar)

On Sturm's Comparison Theorem and Convexity of Zeros

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Since its appearance in 1836 Sturm's Comparison Theorem for second order differential and difference equations has been significantly extended and generalized. One of the most important consequences is the Convexity Theorem about the distance of consecutive zeros. First we show how this can be applied to determine the intervals of convexity and concavity for the zeros of Laguerre, Jacobi and ultraspherical polynomials by a transformation which preserves the zeros. The method also provides lower and upper bounds for the distance between consecutive zeros. Then we present Sturm-type comparison and convexity theorems for second order self-adjoint difference and q-difference equations. These results can be applied to discrete orthogonal polynomials, as an example we show results on the distance of zeros of Hahn and Meixner polynomials.

This is joint work with Kerstin Jordaan from the University of Pretoria and Jemal Gishe from Western Kentucky University.

Factorization in Modular Group and Periodic Negative-Regular Continued Fractions

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The object of the talk is to present properties of periodic continued fractions of the form

$$\underbrace{\frac{-1}{b_1} + \frac{-1}{b_2} + \cdots + \frac{-1}{b_n}}_{n} + \underbrace{\frac{-1}{b_1} + \frac{-1}{b_2} + \cdots + \frac{-1}{b_n}}_{n} + \cdots, \tag{1}$$

where b_k are (positive) integers. This continued fraction is called periodic negative-regular continued fraction. By Tietze's theorem [1], the fraction (1) converges to an irrational number if $|b_k| \geqslant 2$ (except for the case $|b_k| = 2$ for all k).

We have proved that without restriction $|b_k| \geqslant 2$ the fraction (1) may converge to rational numbers or diverge. This is the deference between negative-regular continued fractions and classical regular continued fractions which always converge to irrational numbers.

Several algorithms for construction of periods $\{b_1, \ldots, b_n\}$ of periodic negative-regular continued fractions converging to rational numbers are given. The periods of a given length can be obtained by Fermat's infinite descent method applied to some Diophantine equations. An explicit simple formula for the minimal period for x is presented. A construction using the Calkin-Wilf tree and Stern's diatomic series is described. Arbitrary primitive periods are in one-to-one correspondence with elements of the modular group Γ. Explicit formulas converting products of the standard generators S and ST in Γ into primitive periods are obtained. The periods of elliptic elements of Γ are completely described. This description results in a parametric formula for primitive periods of rational numbers. A periodic negative-regular continued fraction diverges if and only if either its period or its double or its triple represents the identity in Γ.

[1] H. TIETZE, "Uber Kriterien für Konvergenz und Irrationalität unendlicher Ketenbrüche", Math. Ann. **70** (1911).

Multiple Charlier Polynomials: Properties and Asymptotics

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We will introduce multiple Charlier polynomials, which are polynomials which satisfy orthogonality conditions with respect to r Poisson distributions (with different parameters), where $r > 1$. We will give a Rodrigues formula, an explicit expression and a generating function for these polynomials. We also give explicit recurrence relations which hold between neighboring polynomials. We will use this (nearest neighbor) recurrence relations to obtain the asymptotic distribution of the ratio of two neighboring polynomials, which allows us to find the asymptotic distribution of the zeros.

Eigenpolynomials of Dunkl Operators

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Novel classes of orthogonal polynomials that are eigenfunctions of first order differential operators with reflections, i.e. Dunkl operators, have been introduced recently. For a continuous variable they arise as $q = -1$ limit of the little and big q-Jacobi polynomials. In the case of a discrete variable, the Bannai-Ito polynomials, themselves $q = -1$ limits of the q-Racah polynomials enter in this framework as eigenpolynomials of Dunkl shift operators. This talk will review the characterization of these polynomials and their relations.

Based on joint work with Satoshi Tsujimoto, Kyoto University and Alexei Zhedanov, Donetsk Institute for Physics and Technology.

Multidimensional Heisenberg Convolutions and Product Formulas for Multivariate Laguerre Polynomials

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Let p, q positive integers and $M_{p,q}(\mathbb{C})$ the vector space of all complex $p\times q$-matrices. The groups $U_p(\mathbb{C})$ and $U_p(\mathbb{C}) \times U_q(\mathbb{C})$ act on the Heisenberg group

$$H_{p,q} := M_{p,q}(\mathbb{C}) \times \mathbb{R}$$

canonically as groups of automorphisms by multiplication from left and possibly right. The associated orbit spaces may be identified with the spaces $\Pi_q \times \mathbb{R}$ and $\Xi_q \times \mathbb{R}$ respectively with the cone Π_q of positive semidefinite complex matrices of dimension q and the Weyl chamber $\Xi_q = \{x \in \mathbb{R}^q : \ x_1 \geq \ldots \geq x_q \geq 0\}$ of type B_q.

In this talk we present the associated orbit convolutions on $\Pi_q \times \mathbb{R}$ and $\Xi_q \times \mathbb{R}$ explicitly depending on the dimension parameters p. Moreover, we extend these convolutions by analytic continuation to series of convolution structures for arbitrary parameters $p \geq 2q-1$ in both cases. This leads for $q \geq 2$ to continuous series of noncommutative convolutions on $\Pi_q \times \mathbb{R}$ and to commutative convolutions on $\Xi_q \times \mathbb{R}$. In the latter case, we describe the multiplicative functions (often also called hypergroup characters) in terms of multivariate Laguerre and Bessel functions on Π_q and Ξ_q. In particular, we obtain a non-positive product formula for these Laguerre functions on the Weyl chamber Ξ_q. We remark that these Laguerre and Bessel functions on Ξ_q are precisely those which appear in Dunkl-theory of type B_q where one parameter is equal to 1 and the second one has a continuous range.

The results of the talk extend the known case $q = 1$ due to Koornwinder, Trimeche, and others (see [T] and references there) as well as the group case with integers p due to Faraut and Benson et al. (see [F], [BJRW] and references there). Moreover, the results are closely related to product formulas for multivariate Bessel functions and other hypergeometric functions of Rösler who considered the case where the groups $U_p(\mathbb{F})$ and $U_p(\mathbb{F}) \times U_q(\mathbb{F})$ act on the additive groups $M_{p,q}(\mathbb{F})$ for the fields $\mathbb{F} = \mathbb{R}, \mathbb{C}, \mathbb{H}$. Here Röslers approach leads to three continuous series of multivariate Bessel convolutions on the cones $\Pi_q(\mathbb{F})$ of positive semidefinite matrices and the Weyl chamber Ξ_q respectively. We shall also review this case in the talk briefly.

References

[BJRW] C. Benson, J. Jenkins, G. Ratcliff, T. Worku, Spectra for Gelfand pairs associated with the Heisenberg group. *Coll. Math.* 71 (1996), 305–328.

[F] J. Faraut, Asymptotic spherical analysis on the Heisenberg group. *Coll. Math.* 118 (2010), 233–258.

[R] M. Rösler, Bessel convolutions on matrix cones, *Compos. Math.* 143 (2007), 749–779.

[T] K. Trimeche, Generalized Wavelets and Hypergroups. Gordon and Breach, 1997.

[V] M. Voit, Multidimensional Heisenberg convolutions and product formulas for multivariate Laguerre polynomials, preprint 2010.

Orthogonal Polynomials and Expansions for a Family of Weight Functions in Two Variables

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Orthogonal polynomials for a family of weight functions on $[-1,1]^2$,

$$W_{\alpha,\beta,\gamma}(x,y) = |x+y|^{2\alpha+1}|x-y|^{2\beta+1}(1-x^2)^\gamma(1-y^2)^\gamma,$$

are studied and shown to be related to the Koornwinder polynomials defined on the region bounded by two lines and a parabola. In the case of $\gamma = \pm 1/2$, an explicit basis of orthogonal polynomials is given in terms of Jacobi polynomials and a closed formula for the reproducing kernel is obtained. The latter is used to study the convergence of orthogonal expansions for these weight functions. This family of weight functions also admits minimal cubature rules when $\gamma = \pm 1/2$.

Recent Applications of the Kontorovich-Lebedev Transform in Fourier Analysis

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By using integral representations and asymptotic properties of the Kontorovich-Lebedev transform we describe a class of Rajchman measures (whose Fourier-Stieljtes transforms vanish at infinity), which involves the famous Minkowski question mark function (Salem's problem).

Pfaffian Decomposition and a Pfaffian Analogue of q-Catalan Hankel Determinant

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Motivated by the Hankel determinant evaluation of moment sequences, we study a kind of Pfaffian analogue evaluation. We prove an LU-decomposition analogue for skew-symmetric matrices, called Pfaffian decomposition. We then apply this formula to evaluate Pfaffians related to some moment sequences of classical orthogonal polynomials. In particular we obtain a product formula for a kind of q-Catalan Pfaffians. We also establish a connection between our Pfaffian formulas and certain weighted enumeration of shifted reverse plane partitions. This is a joint work with M. Ishikawa and H. Tagawa.

On the Modular Behaviour of the Infinite Product $(1-x)(1-xq)(1-xq^2)(1-xq^3)\cdots$

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Let $q = e^{2\pi i\tau}$, $\Im\tau > 0$, $x = e^{2\pi i\xi} \in \mathbf{C}$ and $(x;q)_\infty = \prod_{n\geq 0}(1-xq^n)$. Let $(q,x) \mapsto (q^*, \iota_q x)$ be the classical modular substitution given by $q^* = e^{-2\pi i/\tau}$ and $\iota_q x = e^{2\pi i\xi/\tau}$. In this talk, we will discuss the "modular behaviour" of the infinite product $(x;q)_\infty$, this means, to compare the function defined by $(x;q)_\infty$ with that given by $(\iota_q x; q^*)_\infty$. Inspired by a work of Stieltjes on some semi-convergent series, we are led to a "closed" analytic formula for $(x;q)_\infty$ by means of the dilogarithm combined with a Laplace type integral that admits a divergent series as Taylor expansion at $\log q = 0$. Thus, we can obtain an expression linking $(x;q)_\infty$ to its modular transform $(\iota_q x; q^*)_\infty$ and which contains, in essence, the modular formulae known for Dedekind's eta function, Jacobi theta function and also for certain Lambert series. Among other applications, one can remark that these results allow to obtain a Ramanujan's asymptotic formula about $(x;q)_\infty$ for $q \to 1^-$.

Elementary Asymptotics of $\Gamma_q(z)$ as $q \to 1$

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In this talk we present two extremely elementary methods to prove the main asymptotic behavior of $\Gamma_q(z)$ as $q \to 1$, using nothing from complex analysis or well-known powerful summation formulas.

Painlevé XXXIV Asymptotics of Orthogonal Polynomials for the Gaussian Weight with a Jump at the Edge

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We study the uniform asymptotics of the polynomials orthogonal with respect to analytic weights with jump discontinuities on the real axis, and the influence of the discontinuities on the asymptotic behavior of the recurrence coefficients. The Riemann-Hilbert approach, also termed the Deift-Zhou steepest descent method, is used to derive the asymptotic results. We take as an example the perturbed Gaussian weight $w(x) = e^{-x^2}\theta(x - \mu_n)$, $x \in \mathbb{R}$, where $\theta(x)$ takes value 1 for $x < 0$, and a nonnegative complex constant ω elsewhere, and $t = 2n/\mu_n^2 \to 1$ as $n \to \infty$. That is, the jump occurs at the edge of the support of the equilibrium measure. The derivation is carried out in the sense of a double scaling limit, namely, $\lim_{n\to\infty}(t-1)n^{2/3} \in \mathbb{R}$ and $n \to \infty$. A crucial local parametrix at the edge point where the jump occurs, is constructed out of a special solution of the Painlevé XXXIV equation. As a main result, we prove asymptotic formulas of the recurrence coefficients in terms of a special Painlevé XXXIV transcendent under the double scaling limit. The special thirty fourth Painlevé transcendent is shown free of poles on the real axis. A consistency check is made with the reduced case when $\omega = 1$, namely the Gaussian weight: the polynomials in this case are the classical Hermite polynomials. A comparison is also made of the asymptotic results for the recurrence coefficients between the case when the jump happens at the edge and the case with jump inside the support of the equilibrium measure. The comparison provides a formal asymptotic approximation of the Painlevé XXXIV transcendent at positive infinity.

An Interpretation of q-Fourier Transform.

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We give a new interpretation of q-Fourier transform from the point of view of the class field theory. This interpretation leads to some conjectures and generalizations about q-Fourier transform.