Experimental investigation of reflection in guided wave-based inspection for the characterization of pipeline defects

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1. Introduction

Pipelines represent crucial infrastructure in the oil, gas, chemical and water transport industries. Significant research has been conducted on topics related to pipeline inspection to meet the continuous requirements of civil and industrial users. To accurately and efficiently carry out planned maintenance and replacement operations on pipelines, the ability to characterise existing or developing defects during pipeline inspection is important for the practical application of non-destructive testing (NDT) techniques. Many of the methods currently available are capable only of providing a qualitative assessment of pipeline defects due to various limitations, such as a low resolution. The use of ultrasonic-guided waves is a new and advanced technique in the NDT field and, thanks to its continuous development in both theory and practice, it has become increasingly attractive for pipeline inspection.

The advantages of guided waves, which include a high degree of sensitivity and versatility, offer great potential for the detection of various defects, but many difficulties in interpreting the wave data collected for accurate defect characterization remain. The complexities that must be considered when using guided wave-based applications relate to wave excitation and propagation as well as interaction with the defect. First, multiple modes are often generated in each guided wave excitation. The resulting wave signals present complicated waveforms because of the existence of the unwanted modes. The important parameters related to defect characterization, such as peak amplitude, peak location and the arrival time of each mode, are consequently difficult to identify. Second, guided waves are inherently dispersive, which causes the wave packet of the mode to spread in space and time after it propagates over a certain distance. This continuous wave distortion renders many of the available signal-processing analysis tools, such as wavelet analysis, insufficiently accurate. Third, the details of the interaction of the guided waves with the discontinuities in the pipeline are not sufficiently elaborate because of the extremely complicated wave properties of guided waves. This interaction may exhibit unknown features because of the interference from the various signal components involved. Therefore, to ensure the accuracy of defect characterization, the problems inherent in the guided wave technique must be carefully considered. Proper control on transduction and excitation of guided waves could help to solve the first two aforementioned problems, so the study reported herein primarily takes into account the third that concerns the defect reflection.

The reflection signal that results from the interaction of the propagating guided waves with the defect in pipeline under examination, in principle, includes substantial defect-relevant information. This information may indicate the presence, location, severity and/or other features of the defect. Better understanding of the details of wave reflection is always helpful to characterise these features of defect. This paper thus aims to investigate this...
Defect characterization-related topics have stimulated a great deal of interest in guided waves-based pipeline research and applications [7–9]. The defect that exists in a pipeline can be approximately represented as a rectangular shape considering the loss of material from pipeline in three dimensions, as shown in Fig. 1. The geometric parameters of a defect are, accordingly, described in terms of radial depth and circumferential and axial extent in a simplified manner. It is believed that a defect can be characterised by identifying and quantifying the effects of its geometric parameters on the reflection signal. Many researchers have investigated the related issues through laboratory experiments and finite element (FE) simulations. For example, Alleyne et al. [7] and Demma et al. [10], respectively, reported that the reflection of incident modes \( L(0,2) \) and \( T(0,1) \) are close to the linear functions of the circumferential extent and radial depth of a defect. Bai et al. [11] used an efficient numerical procedure for the three-dimensional reflection problem to analyze how the length and depth of a circumferential defect affect the scattering characteristics in a pipe. They indicated that the change in the reflection coefficient with increasing circumferential crack length \( L \) is linear to \( L \) when the incident modes consider \( L(0,2) \) and \( T(1,3) \).

Cawley et al. [1] also considered the effect of the axial extent and concluded that the reflection from a defect is a strong function of that defect’s axial extent. The findings of these studies provide a useful foundation for further research. Although these previous research has identified reflection as important for defect characterization, and found that a reflected wave is strongly correlated with the geometric parameters of the defect, the problem of determining the size or severity of pipeline defect has not been figured out yet, especially in a quantitative and accurate manner. The research aiming at developing characterization technique is still ongoing [12].

Demma et al. [10,13] performed substantial parameter study of the reflection from crack and notch in pipeline. They reported that such reflection is the joint result from interference between the reflections at the front and back edges of that defect. It is thus difficult to obtain independent information on the geometric parameters of defect directly from the reflection signal, especially when accurate process models are not available. Although Demma et al. indicates the existence of different components in overall reflection signal, they did not proceed to consider the use of those components for defect characterization, and the majority of published work is still only concerning or studying the overall resulting reflection signal in the inspection-related tasks.

Given all of the difficulties and limitations that exist in the extent research, the purpose of the work presented herein was to propose a new strategy based on the use of embedded reflection components for accurately and quantitatively characterizing pipeline defects in guided wave-based inspection. The respective reflection from front and back edges of defect will be investigated through extensive experiments so as to identify different features of edge reflection signals embedded in the reflection signal, and the relationship between edge reflections and the geometric parameters of defect. Our experimental results were compared and verified with FE simulations each other. The findings will offer insight into more understanding of the reflection of the guided waves from a defect, and accordingly, the new strategy based on edge reflection will be proposed for accurate and quantitative determination of characteristic sizes of pipeline defect.

3. Experimental setup

To identify features of edge reflection signals and reveal the relationships between edge reflections and the defect geometric parameters, experiments were conducted on three same steel pipes that had an external diameter of 34 mm, a wall thickness of 4 mm and a length of 2030 mm. We introduced an artificial notch in each pipe as defect, which had specified axial and circumferential extent and radial depth, as shown in Fig. 2. Over the course of the experiments, the values of these three parameters were gradually increased using a milling machine to simulate defects of different sizes or degrees of severity.

We adopted the longitudinal \( L(0,2) \) mode, as it is easy to excite and has relatively simple acoustic fields [14]. Frequencies ranging from 100 to 240 kHz were chosen because the \( L(0,2) \) mode excited accordingly is non-dispersive, as can be seen from Fig. 3, which shows the phase and group velocity dispersion curves of the pipe under examination. These curves were calculated using the DISPERSE program [15]. The use of the \( L(0,2) \) mode and the frequency range chosen reduced the signal complexity discussed in Section 1 to a certain extent as we considered a single mode and its non-dispersive operating range, thus focusing our investigation on the problem closely relevant to defect reflection.

The experimental setup and instruments used are depicted schematically in Fig. 4. A windowed toneburst consisting of five
cycles at the chosen frequency was delivered through an arbitrary signal generator. The transduction system was carefully designed to excite a single $L(0,2)$ mode into the pipe under examination. A ring consisting of a certain number of piezoelectric transducers (PZTs) was bonded at one end of the pipe to generate and receive guided waves. The PZTs were made of length expander-type piezoelectric material and distributed axisymmetrically, thus ensuring that only the expected longitudinal mode was excited whilst the flexural modes were suppressed. This experimental system is similar to that used in other research\[1,10\]. The defects were initially machined 1300 mm from the end at which the PZTs were bonded. To ensure that the experimental data would have a reliable degree of comparability, each measurement was performed with strictly consistent experimental settings, except for the changes in the defect geometric parameters.

FE-based simulation was conducted in parallel to corroborate our experimental results. The commercial FE program ANSYS was used for the model generation and simulation analysis. The modelled pipes had the same parameters, including material, diameter, thickness and length, as those used in the experiments. The series of defects were introduced into the pipeline model by removing certain elements based on the requirements of the defect position and dimensions set in the experiments. To ensure that the simulation results would be sufficient in terms of accuracy and resolution, we optimised the model parameters and operational setting. The details of this optimisation are reported in a published conference paper\[16\]. To provide a brief example here, Fig. 5 shows the modelled pipe with a 3-mm-wide circumferential defect and the applied loading force as excitation signal.

4. Results and discussion

A series of experiments were performed to investigate the edge reflections of the guided waves created by defects with different geometric parameters. The focus of our analysis here is on the effects of the concerned defect parameter on the reflections at front edge and back edge of defect, respectively, especially the effect of the axial extent, as this parameter was identified as the first important factor in the complexity of reflection.

4.1. Relationship between the edge reflections and the axial extent of a defect

4.1.1. Analysis of experimental results

Tests were performed on a pipe with a notch, the radial depth and circumferential extent of which were kept constant at 21.25% of the pipe wall thickness and 100% of the pipe circumference, respectively. The axial extent was gradually changed from 3 to 170 mm. Data were captured after the manufacture of each defect. While manufacturing the defects according to the proposed sizes, the front edge of each that was nearest to the transducers was kept unchanged, whereas the axial extent to the back edge was increased, as shown in Fig. 6.

Fig. 7 shows the collection of signals reflected from the defect with varying axial extents under excitations at centre frequencies of 145, 175 and 200 kHz, respectively. These three groups of data exhibited a similar trend in the change of the signal pattern. The data collected at the frequency of 175 kHz are used for our subsequent discussion because the defect responses at this frequency were relatively stronger than those at the other frequencies.

The parallel FE simulation was designed to corroborate the effectiveness of the experimental data we had collected. Fig. 8 presents a comparison of the reflection signals resulting from the defects with axial extents of 6, 14, 24 and 42 mm, respectively. The solid lines are the reflection signals obtained from the experiments, and the dashed lines are the FE simulation results. Excellent agreement was demonstrated between them, thus increasing the credibility of both.

Fig. 7 shows that when the axial extent is increased, the wave packet of the reflected signal is extended in time with an increased number of wave cycles and amplitudes of irregular variation. For example, some of the signals in this figure represent the superposition effect in amplitude, whereas others represent
its cancellation. It indicates that the signal that results from the defect reflection involve overlapping signals. These overlapping signals are the result of the reflections at different edges of the defect, a conclusion that was validated in our experiments when the axial extent was sufficiently long (86 and 170 mm in the pipe samples examined). Two separate signals, caused by the front edge and back edge of the defect, were clearly observed over time, and are highlighted with a red line in Fig. 7.

It is interesting to note the coincidence of the same wave pattern in the beginning portion of the collected reflection signals. Fig. 9 presents a plot of the correlation coefficients of reflections of defect at different axial extents, with the reflection of 170-mm-length defect employed as a reference. When the value of the correlation coefficients is 1, the signals at these points have the same pattern. It shows in Fig. 9 that the longer the axial extent, the greater the coincidence of the signals at axial extents with consecutively increasing values. This phenomenon indicates that the signals generated from the front edge of the defect are the same for all defects, regardless of the axial extent. This assumption is further verified by the signals at the axial extents of 86 and 170 mm, as these signals represent the two separate parts of the signal indicated above. The signals reflected from the front edge of the defect at these two axial extents are completely coincident. It further implies that the signal from the back edge of the defect must be complicated as the other part of overall reflection that presents irregular change in Fig. 7.

4.1.2. Front- and back-edge signals embedded in the reflection

As noted in our foregoing discussion, the signal reflected from the entire defect mainly comprises two parts: the signal from the reflection process of the incident waves at the front edge of the defect, and the signal from a more complicated wave interaction process that includes the transmission of waves through the front edge of the defect, reflection from its back edge and the second transmission of the reflected wave through the front edge. Here, we refer to these two parts as the ‘front-edge signal’ and the ‘back-edge signal’. The former primarily depends on the geometric profile of the front edge of the defect, including its depth and circumference. The pattern and energy of the back-edge signal are related to a greater number of factors, including the geometric profile of both edges of the defect and its defective area.

The different features of these two edge signals will inevitably introduce complexity of resulting reflection and difficulty in evaluating defect directly. It should be noted that the effect of reverberations following these two main reflections on the resulting signal can be neglected since they are almost attenuated with high-order factor in the cases where axial extent of defect is not sensitively small with respect to wavelength of propagating waves.

4.1.3. Separation of overlapping front- and back-edge signals

Since the front- and back-edge signals include the information with different features about the characteristics of the defect, the overlapping of these two signals makes the reflection become extremely complex obviously. Therefore, it is useful to extract the two and then analyze individually or jointly in defect characterization. Such an extraction can make the complex reflection problem decomposed into several relatively small problems related to edge reflection. The edge signal involves less defect parameters, so it is relatively easier to be analyzed than the whole reflection problem directly. Moreover, two edge signals can provide more independent and correlated information sources of defect. For example, once the extracted signals have been identified, the axial extent of the defect, which is directly related to the relative distance between two edge reflections, can be easily and accurately derived based on the wave group velocity and the time shifts of the two signals.

We were able to identify the front-edge signal in our experiments by extending the axial extent of the defect (to either 86 or 170 mm), which allows the separate signals from the two edges to be clearly observable over time. Employing this signal as a reference, the back-edge signal embedded in the overall reflection at different axial extents could be extracted simply by subtracting the reference from each reflection signal collected. The results obtained from this process are presented in Fig. 10, which shows that the cycle numbers in all of the back-edge signals remained almost constant and that the amplitudes changed with varying axial extents. Small amount of noise appears in the rear of each signal due to the reverberation between the two edges of defect.

The individual features of the front- and back-edge signals can be further elucidated by the separation results obtained. Both of these signals in the different defect scenarios exhibit nearly the same patterns in terms of their number of cycles, frequency and modulation. The amplitude of the front-edge signal remains the same for all of the scenarios with various axial extents because this signal depends primarily on the front edge of the defect, as previously discussed. The amplitude of the back-edge signal is more complex, presenting different variation trends at different axial extents. Fig. 11 shows the energies of the reflection coefficient spectrum for the back-edge signals at these different
extents. The $Y$ coordinate is the root-mean-square (RMS) value of the reflected back-edge signals, with the RMS value of the common front-edge signal taken as the normalisation factor. The $X$ coordinate is the axial extent to wavelength ratio for the incident wave at 175 kHz (31.157 mm). From Fig. 11(a), it can be seen that the reflection coefficients of the back-edge signal at $X$ from 0 to 1.4 demonstrate a roughly periodical variation due to the effect of the defect’s front edge on the generation of the back-edge signal. With an increase in the axial extent, the reflection coefficients of the back-edge signal exhibit the relatively stable variation depicted in Fig. 11(b). This is because the front- and back-edge signals are completely separate in time in these situations, and then the latter primarily depends on the geometric features of the defect’s back edge. Further, it can also be noted that the reflection coefficients of the front-edge signals are always larger than those of their back-edge counterparts.

We were able to obtain these results easily with our collected experimental data as the reference, namely, that the front-edge signal can be identified when the axial extent of the defect is made long enough. In practical applications, the extraction of the edge reflection components can be achieved by applying data analysis-based method to the collected signal directly. The technique for this problem has been developed by the authors, which will be reported in another forthcoming paper.
4.1.4. Determination of the axial extent of a defect based on the separated signals

The time shift between the front- and back-edge signals corresponds to the time that the guided waves take to propagate between the front and back edges of the defect, namely, twice its axial extent. Thus, once the two edge signals have been identified, the axial extent of the defect can be calculated by the following equation:

\[ L = \frac{D}{C^2} v_{gr} \]  

(1)

where \( L \) is the length of the defect in the axial direction, \( D \) is the time shift between the front- and back-edge signals, and \( v_{gr} \) is the group velocity of the propagating guided waves. An accurate determination of this time shift is the first key to calculating the

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**Fig. 9.** Correlation function curves of defect reflections, with the reflection from a 170 mm length defect employed as a reference.

**Fig. 10.** Extraction of front- and back-edge signals from the experimental reflection signal collected from the defects with varying axial extents, constant radial depths and constant circumferential extents.
As the front- and back-edge signals present a similar pattern in the time domain, the cross-correlation function (CCF) of the two is computed to estimate time shift $D$. In this method, the time shift between the two signals corresponds to the maximum value of their CCF, as shown below.

$$D = \max \left( \frac{1}{T} \int_0^T \! x(t)y(t+\tau) \, dt \right),$$

where $T$ is observation time, and $x$ and $y$ are front- and back-edge signals, respectively.

The estimation results of the time shift ($D$) and the axial extent ($L$) of the defect from the actual measurements and the experimental and simulation data are presented in Tables 1 and 2, from which it can be seen that the experimental and simulation results are very close to the actual measurements. The relationship between the time shift of the two edges of the reflected waves and the axial extent of the defects based on actual measurements, experiments, and simulations are also presented in curve form in Fig. 12, which again demonstrates the excellent agreement among them. Moreover, it was identified from the correlation and comparison processes that the phase of the front-edge signal is the reverse of that of the back-edge signal. These conclusions also validate the acceptability of our extraction operation of the front- and back-edge signals.
4.2. Relationship between the edge reflections and the circumferential extent of a defect

As most pipeline defects are non-axisymmetric in reality, it is also necessary to investigate the relationship between the edge reflections and circumferential extent of a defect. Accordingly, we conducted experiments employing defects that represented six different circumferential extent scenarios: 60/360, 120/360, 180/360, 240/360, 300/360, and 360/360 (as a percentage of the full circumferential extent). The radial depth of the defects was fixed at 1 mm (25% of the pipe wall thickness), and their axial extent was held constant at 170 mm. The use of defect with long axial extent was to ensure that the reflections from the front and back edges would be separate over time themselves so that the effect of the circumferential extent on the respective edge signal can be directly identified.

Our experimental results are shown in Fig. 13. As other researchers have also observed, mode conversion occurs at defects with a non-axisymmetric circumferential extent [18]; that is, other modes are generated in addition to mode $L(0, 2)$. Bai [10] indicated that incident mode $L(0, 2)$ is transmitted with most of the energy in it before and after the reflection. As $L(0, 2)$ is the primary reflection mode, its amplitude is relatively large and clearly distinguishable. To determine the effects of other modes on the mode $L(0, 2)$ reflection at both edges of a defect, the time shift between the modes $L(0, 2)$ of the two observed edge signals was identified employing the CCF method to calculate the axial extent of the defect. The result obtained through this procedure (165.85 mm) is approximately equal to the real value (170 mm), thus verifying that mode $L(0, 2)$ is transmitted with most of the energy in it before and after the reflection. As $L(0, 2)$ is the primary reflection mode, its amplitude is relatively large and clearly distinguishable. To determine the effects of other modes on the mode $L(0, 2)$ reflection at both edges of a defect, the time shift between the modes $L(0, 2)$ of the two observed edge signals was identified employing the CCF method to calculate the axial extent of the defect. The result obtained through this procedure (165.85 mm) is approximately equal to the real value (170 mm), thus verifying that mode $L(0, 2)$ was not interfered with other modes in reflection. The reflection echo of mode $L(0, 2)$ is thus directly used in analysis.

It can be observed from Fig. 13(a) that the time shift of the two edge signals remains constant when the axial extent of the defect remains the same. With an increase in the circumferential extent, the amplitudes of both the front- and back-edge signals can be observed to increase. To present the change in the two reflections more clearly, the curves of reflection coefficients of the edge signals as a function of the circumferential extent of the defect are plotted and shown in Fig. 13(b). Both reflections are normalised by the RMS value of the incident signal. It can be seen that the reflection from the front edge has almost the same change rate as that from the back edge, both of which increase roughly linearly with respect to the circumferential extent of the defect. The reflection coefficient of the back-edge signal is always smaller than that of the front-edge signal in every scenario.

4.3. Relationship between the two edge reflections and the radial depth of the defect

To investigate the relationship between radial depth and the front- and back-edge signals of a defect, we performed experiments on pipe models whose defects extended to the full circumference, had a constant axial extent and varying radial depths. The constant value of the axial extent chosen for this series of testing was 86 mm, at which, as previously discussed, the front- and back-edge signals are ensured to be separate over time so that the effect of radial depth on edge reflections can be clearly revealed. The radial depths considered were 0.85, 1.64, 2.42 and 3.21 mm, which corresponded to 21%, 41%, 61% and 80% of the pipe wall thickness, respectively.

Fig. 14(a) shows the resulting data, and the reflection coefficients of the front- and back-edge signals as a function of the radial depth of the defect are shown in Fig. 14(b). It can be clearly seen that the front-edge signal increases monotonically or roughly linearly with respect to the radial depth. Of particular interest, however, is that the back-edge signal fails to exhibit the same monotonic increase. This means that a defect with a back edge of greater depth can produce a smaller back-edge reflection than will a defect with a back edge of less depth. Such feature of nonmonotonic trend cannot be presented by the overall reflection signal, which always has a rough linear relationship with the change of radial depth as reported by other researchers. We surmise that this is because the transmission energy of the waves that reach the back edge of defect is diminished when the reflection at the front edge of defect becomes strong to a specific value. What can be stated with assurance is that the back-edge signal depends on more factors or parameters than does its...
front-edge counterpart. Moreover, combined with the findings in Section 4.2, it indicates that the circumference extent and radial depth reveal different features in their effects on two edge reflections, which provide the possibility of characterizing these two geometric parameters further through proper decomposition techniques.

4.4. Proposed strategy for the characterization of pipeline defects

Based on the useful observations and findings thus far presented, we here propose a new strategy for the accurate and quantitative characterization of pipeline defects. This strategy primarily involves the extraction of two edge signals from single overall reflection signal and the use of these identified edge signals for defect characterization. As discussed, two parts are embedded in a defect’s reflection signal: front-edge signal and back-edge signal. The patterns of the front- and back-edge signals remain nearly constant for defect scenarios of varying axial and circumferential extent and radial depth. However, their amplitudes demonstrate different features with a continuous increase in a specified defect geometric parameter. The front-edge signal is quite simple, and its amplitude characteristics always change in a linear fashion. The amplitudes of the back-edge signal, in contrast, exhibit periodic, monotonic or nonmonotonic variations with different geometric parameters. Furthermore, it can be concluded that the axial extent of defect is closely related to the phase information of two edge signals while its circumferential extent and radial depth have important effect on their amplitudes in a joint manner. Identifying these edge signals once extracted, together with their phase and amplitude features, can provide sufficient information to enable accurate and quantitative defect characterization.

The axial extent of the defect can be easily determined from the time shift between the two identified edge signals, as this shift in time bears a direct linear relationship with the length of the defect. Each identified edge signal includes geometric information on the corresponding edge of the defect, so the radial depth and circumferential extent of that defect can be determined through introducing proper decomposition and analytical techniques on those edge signals. The results reported in Sections 4.2 and 4.3 have shown great feasibility for this purpose. The further quantitative description of a defect requires a consideration of the attenuation and dispersion of the guided waves during propagation, which allows the refinement of the obtained results. In addition, it is worthy to note that the issues of multiple-mode and dispersion as aforementioned in Section 1 should be avoided as much as possible to simply the overall reflection signal first through a good controlling on transduction and excitation so that the proposed strategy could be implemented better.

Our proposed strategy decomposes the reflection of guided waves at the defect in a very complex form into the relatively simple reflection problems that occur at its front and back edges. Such process obviously reduces the complexity of the concerned problem because the destructive effect of the overlap between two edge signals on reflection can be efficiently removed. At the same time, it provides more information sources about the defect, thus considerably enhancing the reliability and accuracy of quantitative defect characterization. In order to completely achieve the objectives based on this proposed strategy, there are series of follow-up researches that need to be conducted, for example, the extraction of edge reflections and further decomposition of each reflection component.

5. Conclusion

The study reported herein investigated the reflection of the guided waves at the front and back edges of pipeline defect, and particular attention is paid in this work to the respective effects of the geometric parameters of defect on each edge reflection signal. The results show that the complexity of a defect’s reflection signal results from the overlap between the reflected edge signals with different features. Extensive experimentation, corroborated by FE simulation, was performed on artificial defects of various sizes in pipeline for reflection study. A new strategy is accordingly proposed for the accurate and quantitative characterization of pipeline defects in using guided wave-based inspection method. The findings presented in this paper can serve as a foundation for further work on this topic. The notch-type defect considered in this paper involves two obvious changes of cross-sectional area at the boundaries between the defective area and pipe wall, so the front-edge signal and back-edge signal can be naturally identified as primary components of overall reflection signal to be extracted.
for defect characterization. The proposed strategy can be extended to more complicated defects or real situations since it appears as a simplification method for complex problem based on the extraction of reflection components. The defective boundaries of any defect will be taken as the features related to size or severity information of that defect, which generate the strong reflections to overlap in overall reflection signal. The complex length profile of defect will not have a significant effect on reflection signal since it only involves the tapers extending over much small height or distant. That is, the overall reflection signal from the defect will mainly include the reflection components caused by the boundaries of defective area of concerned defect. In the frame of proposed strategy, the destructive effect of axial extent on reflection can be eliminated through the extraction of boundary reflection components. Although each extracted component in practical cases will be related to more characteristic parameters of defect besides the circumferential extent and radial extent as discussed in this paper, its complexity is lower certainly compared to the overall reflection signal. Therefore, it is expected that the strategy proposed herein will enable or facilitate the accurate and quantitative characterization of defect parameters, especially the axial extent of defect. This strategy needs to be refined when the defect is very small since the reverberations following the first back-edge reflection have to be considered.

In general, the interaction of the guided waves of a real defect, such as corrosion, is a relatively more complicated phenomenon due to propagation in the rough defective area and reflection of waves at the irregular edges. Such practical problems require further research to identify their physical mechanisms from a theoretical perspective and to generalise the results of this research to more situations.

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