Importance sampling methods are proposed to estimate joint default probabilities under credit risk models of structural form and reduced form. By means of large deviation theory, the proposed importance sampling has a virtue of the minimal sample variance, which means our method is efficient. When random environment such as stochastic correlation/volatility arises, asymptotic expansion obtained from singular perturbation is found useful to combine with large deviation techniques in order to estimate default probabilities. Applications of pricing credit derivatives, risk management of credit portfolio, and a comparison with a Matlab program are demonstrated in this talk.