



## DEPARTMENT OF MATHEMATICS

City University of Hong Kong

# Lagrangian Uncertainty Quantification and Information Inequalities for Stochastic Flows

by

**Prof. Michal BRANICKI**  
*The University of Edinburgh, UK*

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**Time: 4:00pm – 5:00pm**

**Venue: B5-308, Yeung Kin Man Academic Building**

### ABSTRACT

We develop a systematic information-theoretic framework for quantification and mitigation of error in probabilistic Lagrangian (i.e., path-based) predictions which are obtained from dynamical systems generated by uncertain (Eulerian) vector fields. This work is motivated by the desire to improve Lagrangian predictions in complex dynamical systems based either on analytically simplified or data-driven models. We derive a hierarchy of general information bounds on uncertainty in estimates of statistical observables  $E^{\{\nu\}}[\text{ff}]$ , evaluated on trajectories of the approximating dynamical system, relative to the 'true' observables  $E^{\{\mu\}}[\text{ff}]$  in terms of certain  $\varphi$ -divergencies,  $D_f(\mu \setminus \nu)$  which quantify discrepancies between probability measures  $\mu$  associated with the original dynamics and their approximations  $\nu$ . We then construct scalar fields of finite-time divergence rates which are rooted in information theory/geometry, and we show their existence and space-time continuity for general stochastic flows. Combining these divergence rate fields with information inequalities allows for a principled quantification of Lagrangian uncertainty in a given dynamics, as well as a mitigation of uncertainty in path-based observables estimated from simplified models of the true dynamics in a way that is amenable to algorithmic implementations. We also derive a link between the divergence rates and so-called finite-time Lyapunov exponents for probability measures and for path-based observables.

~ALL ARE WELCOME~

