Workshop on Conservation Laws
Date: 19-20 December 2023
Venue: P4704

Day 1 --- 19 December 2023 (Tuesday)

8:45 am - 9:10 am	**WELCOME and OPENING**
9:10 am - 10:10 am	Session Chair: Prof. Tong YANG
9:10 am - 9:40 am	**Prof. Feimin HUANG** Compensated compactness and hyperbolic conservation laws
9:40 am - 10:10 am	**Prof. Beixiang FANG** On uniqueness of steady 1-D shock solutions in a finite nozzle via vanishing viscosity arguments
10:10 am - 10:30 am	**COFFEE BREAK**
10:30 am - 12:00 am	Session Chair: Prof. Dehua WANG
10:30 am - 11:00 am	**Prof. Xiaozhou YANG** Multi-shock wave in multi-dimensional conservation laws
11:00 am – 11:30 am	**Prof. Hyangdong PARK** Transonic shocks for three-dimensional axisymmetric flows in divergent nozzles
11:30 am – 12:00 am	**Prof. Qin WANG** Some new possible structures for shock regular reflection by straight wedges
12:00 am - 2:00 pm	**LUNCH BREAK**
2:00 pm - 3:30 pm	Session Chair: Prof. Xiaozhou YANG
2:00 pm - 2:30 pm	**Prof. Yi WANG** Vanishing viscosity limit and time-asymptotic stability of planar rarefaction wave to multi-dimensional compressible Navier-Stokes equations
2:30 pm - 3:00 pm	**Prof. Jun CHEN** Stability of Transonic Shocks past 3-D Wedges
3:00 pm – 3:30 pm	**Prof. Jie KUANG** Some mathematical analysis on hypersonic similarity
3:30 pm - 3:50 pm	**COFFEE BREAK**
3:50 pm - 5:50 pm	Session Chair: Prof. Xianpeng HU
3:50 pm - 4:20 pm	**Prof. Tian-Yi WANG** Isothermal Limit of Entropy Solutions of the Euler Equations for Isentropic Gas Dynamics
4:20 pm - 4:50 pm	**Prof. Changguo XIAO** Hydrodynamic limit and Newtonian limit from the relativistic Boltzmann equation to the classical Euler equations
4:50 pm – 5:20 pm	**Prof. Feng XIAO** Dynamical stability of steady normal shock structures for potential flow
5:20 pm – 5:50 pm	**Dr. Yun PU** An Inverse Problem for Supersonic Potential Flows Past a Cone

Day 2 --- 20 December 2023 (Wednesday)

8:40 am - 10:10 am	Session Chair: Prof. Feimin HUANG
8:40 am- 9:10 am	**Prof. Yaguang WANG** Study of boundary layers in geophysical flow
9:10 am - 9:40 am	**Prof. Tianhong LI** An analysis to a model of tornado
9:40 am - 10:10 am	**Prof. Xuemei DENG** Elliptic Equation in Divergence Form with Discontinuous Coefficients in Domains with Corners
10:10 am - 10:30 am	**COFFEE BREAK**
10:30 am - 12:00 am	Session Chair: Prof. Yaguang WANG
10:30 am - 11:00 am	**Prof. Yongqian ZHANG** On an inverse problem to determine the shape of bending wall in the supersonic flow
11:00 am – 11:30 am	**Prof. Gaowei CAO** New Formula for Entropy Solutions for Scalar Hyperbolic Conservation Laws: Nonuniform Convexity of Flux Functions and Fine Properties of Solutions
11:30 am – 12:00 am	**Prof. Siran LI** The isometric immersions problem: from perspectives of PDE, geometry, and physics
12:00 am - 2:00 pm	**LUNCH BREAK**
2:00 pm - 3:30 pm	Session Chair: Prof. Yongqian ZHANG
2:00 pm - 2:30 pm	**Prof. Yong WANG** Global Solutions of the Compressible Euler and Euler-Poisson Equations with Large Initial Data of Spherical Symmetry
2:30 pm - 3:00 pm	**Prof. Qin ZHAO** Asymptotic analysis of transonic shocks in divergent nozzles with respect to the expanding angle
3:00 pm – 3:30 pm	**Dr. Song LIU** Two-dimensional Riemann problem with four-shock interactions for the Euler equations for potential flow
3:30 pm - 3:50 pm	**COFFEE BREAK**
3:50 pm - 5:50 pm	Session Chair: Prof. Wei XIANG
3:50 pm - 4:20 pm	**Prof. Tao WANG** Stability of characteristic discontinuities in thermoelasticity
4:20 pm - 4:50 pm	**Ms. Jianing YANG** Characterizations for the depletion of reactant in a one-dimensional dynamic combustion model

Gaowei Cao (Innovation Academy for Precision Measurement Science and Technology, CAS)

Title: New Formula for Entropy Solutions for Scalar Hyperbolic Conservation Laws: Nonuniform Convexity of Flux Functions and Fine Properties of Solutions

Abstract: We are concerned with the Cauchy problem of $1D$ scalar hyperbolic conservation laws with flux functions of nonuniform convexity and the initial data functions in L^∞. We first present and prove a new formula for entropy solutions for the Cauchy problem with flux functions of strict convexity, which generalizes the Lax-Oleinik formula that requires the flux function to be uniformly convex. As the applications of the new formula for entropy solutions, we prove fine properties of entropy solutions particularly including the following:

1. Classification of basic waves in the solutions for the Cauchy problem.
2. Fine structures of backward characteristic triangles, directional limits of entropy solutions, general structures of shock curves, classification of discontinuous points, priori and exact lifespan of characteristics, and formation and development of shock waves.
3. Invariants, complete description of divides of entropy solutions, and global structures on entropy solutions.
4. Asymptotic behaviors of entropy solutions for L^∞ initial data functions in both L^∞-norm and L^p_{loc}-norm: the asymptotic profile in L^∞-norm is a shock-free solution or a single shock wave, and in L^p_{loc}-norm is the generalized N-wave.

This is a joint work with Prof. Gui-Qiang G. Chen and Xiaozhou Yang.

Jun Chen (China Three Gorges University)

Title: Stability of Transonic Shocks past 3-D Wedges

Abstract: I will talk about the stability of three-dimensional transonic shocks governed by the 3-D potential flow equation. It is showed that for a piecewise constant weak transonic flow, if the incoming flow and the wedge are slightly perturbed, there exist a unique weak transonic shock and downstream subsonic solution, which is also a small perturbation from the background solution. The connection between the shock condition and the elliptic estimates will be explained. I will also introduce a recent result about the stability of strong transonic shocks over 3-D wedges. We show that the strong shocks near normal shock regime are stable.

Xuemei Deng (China Three Gorges University)

Title: Elliptic Equation in Divergence Form with Discontinuous Coefficients in Domains with Corners

Abstract: We study the equations in divergence form with piecewise C^{α} coefficients. The domains contain corners, and the discontinuity surfaces are attached to edges of the corners. We obtain piecewise $C^{1,\alpha}$ estimates across the discontinuity surfaces and provide an example to illustrate the issue about the regularity at the corners.

Beixiang Fang (Shanghai Jiao Tong University)

Title: On uniqueness of steady 1-D shock solutions in a finite nozzle via vanishing viscosity arguments

Abstract: In this talk, I will report our recent results on uniqueness of steady 1-D shock solutions in a finite nozzle via vanishing viscosity arguments. It is well-known from the viewpoint of inviscid flows that for a given supersonic state at the entrance of the nozzle, there exist infinite transonic shock solutions with the same state behind the shock front, while the position of the shock front could be arbitrary in the nozzle. In this talk, we are going to investigate the uniqueness of the inviscid shock solution by regarding it as a vanishing viscosity limit of smooth viscous shock solutions for the steady 1-D Navier-Stokes system. It will be shown that the viscous shock solutions converge under the L^1 norm as the viscosity coefficient goes to zero, which implies the uniqueness of the steady 1-D shock solution for the inviscid flow. Moreover, the position of the shock front for the limit shock solution can also be obtained. This is a joint work with Dr. Qin Zhao.

Feimin Huang (Academy of Mathematics and Systems Science, Chinese Academy of Sciences)

Title: Compensated compactness and hyperbolic conservation laws

Jie Kuang (Innovation Academy for Precision Measurement Science and Technology, CAS)

Title: Some mathematical analysis on hypersonic similarity

Abstract: In this talk, I will present our recent results on the mathematical validation of hypersonic similarity law for steady hypersonic flow over two-dimensional Lipschitz wedge and the convergence rates are also present for both potential flow and full Euler equations under the frame work of BV∩L 1 . This talk is based on the joint works with Prof. Gui-Qiang G. Chen, Prof. Wei Xiang and Prof. Yongqian Zhang.

Tian-Hong Li (Academy of Mathematics and Systems Science, Chinese Academy of Sciences)

Title: An analysis to a model of tornado

Abstract: Tornado is a destructive catastrophe. We use compressible isentropic Euler equations to describe this problem.

Siran Li (Shanghai Jiao Tong University)

Title: The isometric immersions problem: from perspectives of PDE, geometry, and physics

Abstract: We report our recent work on a classical problem in differential geometry: isometric immersions and/or embeddings of Riemannian and semi-Riemannian manifolds. The underlying PDE is the system of Gauss-Codazzi-Ricci equations. Existence of isometric immersions is studied under various curvature conditions, via elliptic and hyperbolic PDE techniques. Weak continuity of isometric immersions is investigated with the help of the theory of compensated compactness. Connections to other problems in mathematical physics, including fluid dynamics, harmonic maps, and nonlinear elasticity, will be discussed.

This project was initiated during my D.Phil. studies under the supervision of Prof. Gui-Qiang Chen, and most of the developments reported today are established in collaboration with and/or influenced by Prof. Chen.

Song Liu (Hong Kong Polytechnic University)

Title: Two-dimensional Riemann problem with four-shock interactions for the Euler equations for potential flow.

Abstract: We present a rigorous approach and related techniques to construct global solutions of the 2-D Riemann problem with four-shock interactions for Euler equations for potential flow. The 2-D Riemann problem is reformulated to a shock reflection-diffraction problem with respect to a symmetric line. After introducing three critical angles: the vacuum critical angle from the compatibility conditions, the detachment angle and the sonic angle, we clarify all configurations of the Riemann solutions for the interactions of two-forward and two-backward shocks. Then the problem is further reformulated to the free boundary value problem of a second-order quasilinear equation of mixed elliptic-hyperbolic type in a pseudo-subsonic domain, along with two sonic boundaries varying with the choice of two independent incident angles. The difficulties arise from the degenerate ellipticity near the sonic boundaries, the nonlinearity of the free boundary condition, and the singularity of the solution near the corners of the domain. To solve the problem, we need to analyze the solutions for a quasilinear degenerate elliptic equation by the maximum principle of the mixed-boundary value problem, the theory of the oblique derivative problem, the uniform a priori estimates, and the iteration method. This talk is based on a joint work with Gui-Qiang Chen, Feimin Huang, Alex Cliffe and Qin Wang.

Hyangdong Park (Korea Institute for Advanced Study)

Title: Transonic shocks for three-dimensional axisymmetric flows in divergent nozzles

Abstract: We prove the stability of three-dimensional axisymmetric solutions to the steady Euler system with transonic shocks in divergent nozzles under perturbations of the exit pressure and the supersonic solution in the upstream region. We first derive a free boundary problem with the newly introduced formulation of the Euler system for three-dimensional axisymmetric flows in divergent nozzles via the method of Helmholtz decomposition. We then construct an iteration scheme and use the Schauder fixed point theorem and weak implicit function theorem to solve the problem.

Yun Pu (Academy of Mathematics and Systems Science, Chinese Academy of Sciences)

Title: An Inverse Problem for Supersonic Potential Flows Past a Cone

Abstract: In this talk, we try to establish the existence of entropy solutions to the inverse problems for three-dimensional axisymmetric supersonic potential flows past a cone with a relatively large vertex angle. In this inverse problem, the incoming flow and the pressure distribution on the surface of an axisymmetric cone are prescribed, and we are required to find the generating curve of the cone and to determine the surrounding flow field. A modified Glimm scheme is applied to solve this inverse problem and the asymptotic behaviours of the entropy solution, the generating curve, and the strong shock are also studied.

Qin Wang (Yunnan University)

Title: Some new possible structures for shock regular reflection by straight wedges

Abstract: We are concerned with the shock regular reflection configurations of unsteady global solutions for a plane shock hitting a symmetric straight wedge. It has been known that the patterns of shock reflection are various and complicated, including regular and Mach reflection. Most of the fundamental issues for shock reflection have not been understood. Recently there are great progress on the mathematical theory of shock regular reflection problem, especially for the global existence, uniqueness and structural stability of solutions. In this talk, we will show that there are two more possible configurations of shock regular reflection besides known four configurations.

Tao Wang (Wuhan University)

Title: Stability of characteristic discontinuities in thermoelasticity
Abstract: The system of thermoelasticity governing the evolution of thermoelastic nonconductors of heat allows two types of characteristic discontinuities: thermoelastic contact discontinuities for which the velocity is continuous across the discontinuity interface, and thermoelastic vortex sheets for which the tangential velocity can undergo a jump. In this talk we will present the results on linear stability of thermoelastic contact discontinuities (joint with Gui-Qiang G. Chen and Paolo Secchi) and nonlinear stability of elastic vortex sheets (joint with Robin Ming Chen, Jilong Hu, Dehua Wang, and Difan Yuan).

Tian-Yi Wang (Wuhan University of Technology)

Title: Isothermal Limit of Entropy Solutions of the Euler Equations for Isentropic Gas Dynamics

Abstract: In this talk, we want to present the isothermal limit of entropy solutions in L^∞, containing the vacuum states, of the Euler equations for isentropic gas dynamics. First, We want to start with the explicit asymptotic analysis of the Riemann solutions containing the vacuum states. Then, we want to show the entropy solutions in L^∞ of the isentropic Euler equations converge strongly to the corresponding entropy solutions of the isothermal Euler equations, when the adiabatic exponent $\gamma \rightarrow 1$. This is achieved by combining careful entropy analysis and refined kinetic formulation with compensated compactness argument to obtain the required uniform estimates for the limit. The entropy analysis involves careful estimates for the relation between the corresponding entropy pairs for the isentropic and isothermal Euler equations when the adiabatic exponent $\gamma\to 1$. The kinetic formulation for the entropy solutions of the isentropic Euler equations with the uniformly bounded initial data is refined, so that the total variation of the dissipation measures in the formulation is locally uniformly bounded with respect to $\gamma>1$. This is the joint work with Gui-Qiang G. Chen, and Fei-Min Huang.

Yaguang Wang (Shanghai Jiao Tong University)

Title: Study of boundary layers in geophysical flow

Abstract: In this talk, we shall review the mathematical analysis of boundary layers in geophysical flow in the large Reynolds number and beta-plane parameter limit, including the behavior of boundary layers, the well-posedness of boundary layer equations, and the justification of the boundary layer expansions.

Yi Wang (Academy of Mathematics and Systems Science, Chinese Academy of Sciences)

Title: Vanishing viscosity limit and time-asymptotic stability of planar rarefaction wave to multi-dimensional compressible Navier-Stokes equations

Abstract: The talk is concerned with our recent several works on vanishing viscosity limit and time-asymptotic stability of planar rarefaction wave to multidimensional (2D/3D) compressible isentropic/full Navier-Stokes equations, which are directly motivated by Professor Gui-Qiang G. Chen and his collaborator's work on the uniqueness of planar rarefaction wave to the planar Riemann problem of multi-dimensional compressible Euler equations. If time permits, these topics on other planar wave patterns, i.e., planar contact discontinuity and planar viscous shock wave will also be involved.

Yong Wang (Academy of Mathematics and Systems Science, Chinese Academy of Sciences)

Title: Global Solutions of the Compressible Euler and Euler-Poisson Equations with Large Initial Data of Spherical Symmetry

Abstract: In this talk, we are concerned with the global existence theory for finite-energy solutions of the multidimensional compressible Euler equations and Euler-Poisson equations (both gaseous stars and plasmas are included) with large initial data of spherical symmetry. One of the main challenges is the strengthening of waves as they move radially inward towards the origin, especially under the self-consistent gravitational field for gaseous stars. A fundamental unsolved problem is whether the density of the global solution forms a delta measure (i.e., concentration) at the origin. We develop a new approach for the construction of approximate solutions as the solutions of an appropriately formulated problem for the compressible Navier-Stokes(-Poisson) equations with a carefully adapted class of degenerate density-dependent viscosity terms, so that a rigorous convergence proof of the approximate solutions to the corresponding global solution of the compressible Euler equations and Euler-Poisson equations with large initial data of spherical symmetry can be obtained. Even though the density may blow up near the origin at a certain time, it is proved that no delta measure (i.e., concentration) in space-time is formed in the vanishing viscosity limit for the finite-energy solutions of the compressible Euler-Poisson equations for both gaseous stars and plasmas in the physical regimes under consideration. The talk is based on joint works with G.Q. Chen, F.M. Huang, T.H. Li, L. He, W.Q. Wang, D.F. Yuan.

Changguo Xiao (Guangxi Normal University)

Title: Hydrodynamic limit and Newtonian limit from the relativistic Boltzmann equation to the classical Euler equations

Abstract: We justify rigorously the validity of the two independent limits from the special relativistic Boltzmann equation to the classical Euler equations without assuming any dependence between the Knudsen number ε and the light speed $\mathfrak{c}$. The convergence

rates are also obtained. This is achieved by Hilbert expansion of relativistic Boltzmann equation. New difficulties arise when tacking the uniform in $\mathfrak{c}$ and ε estimates for the Hilbert expansion, which have been overcome by establishing some uniform-in-$\mathfrak{c}$ estimates for relativistic Boltzmann operators.

Feng Xiao (Hunan Normal University)

Title: Dynamical stability of steady normal shock structures for potential flow
Abstract: In this talk, I will talk about the dynamical stability problem of steady normal shocks for potential flows in 2D and 3D. The dynamical stability problem will be formulated as an initial boundary value problem of non-linear hyperbolic equations of second order in non-smooth domain. Due to the non-smoothness of the space-domain, it is difficult for us to establish the well-posedness of the linearized problem and the energy estimates. By introducing proper extension method and modified partial hodograph transform, we are able to overcome these difficulties and the dynamical stability of the steady normal shocks can be established. This is based on works joint with Prof. Beixiang Fang, Prof. Feimin Huang and Prof. Wei Xiang.

Jianing Yang (Shanghai Jiao Tong University)

Title: Characterisations for the depletion of reactant in a one-dimensional dynamic combustion model

Abstract: In this talk, a novel observation is made on a one-dimensional compressible Navier—-Stokes model for the dynamic combustion of a reacting mixture of γ-law gases ($\gamma>1$) with discontinuous Arrhenius reaction rate function, on both bounded and unbounded domains. We show that the mass fraction of the reactant (denoted as Z) satisfies a weighted gradient estimate $Z_y/sqrt{Z}\in L_t^{\infty}L_y^2$, provided that at time zero the density is Lipschitz continuous and bounded strictly away from zero and infinity. Consequently, the graph of Z cannot form cusps or corners near the points where the reactant in the combustion process is completely depleted at any instant, and the entropy of Z is bounded from above.

Xiaozhou Yang (Innovation Academy for Precision Measurement Science and Technology, CAS)

Title: Multi-shock wave in multi-dimensional conservation laws.

Yongqian Zhang (Fudan University)

Title: On an inverse problem to determine the shape of bending wall in the supersonic flow.

Qin Zhao (Wuhan University of Technology)

Title: Asymptotic analysis of transonic shocks in divergent nozzles with respect to the expanding angle

Abstract: Transonic shocks play a pivotal role in designation of supersonic inlets and ramjets. It is well-known that there exist infinite shock solutions for steady 1-D flows in a flat nozzle with the position of the shock front being arbitrary, while there exists a unique shock solution in a divergent nozzle as the pressure at the exit is given within an appropriate interval. In this talk, we

focus on the asymptotic behaviour of the transonic shock solutions in divergent nozzles as the expanding angle goes to zero, and try to figure out a criterion which may be used to select the physical one among all shock solutions in the flat nozzle. The talk is based on joint work with Prof. Beixiang Fang, Dr. Xin Gao.