Liu Bie Ju Centre for Mathematical Sciences
City University of Hong Kong

Mathematical Analysis and its Applications Colloquium

Organized by Prof. Ya Yan LU and Prof. Wei Wei SUN

The Lagrange method and SAO with bounds on the dual variables

by

Professor M.J.D. Powell
University of Cambridge, UK

Date : Nov 16, 2011 (Wednesday)
Time : 4:30 pm to 5:30 pm
Venue: Room B6605 (College Conference Room)
Blue Zone, Level 6, Academic 1 (AC1)
City University of Hong Kong

ABSTRACT:

In the Lagrange method for constrained optimization, estimates of the Lagrange multipliers (dual variables), $\underline{\lambda} \in \mathcal{R}^m$ say, are adjusted in an outer iteration, and the Lagrange function $L(\underline{x}, \underline{\lambda})$, $\underline{x} \in \mathcal{X}$, is minimized for each $\underline{\lambda}$, where $\underline{x} \in \mathcal{R}^n$ is the vector of primal variables, and where $\mathcal{X}$ is a prescribed compact subset of $\mathcal{R}^n$. Let $\phi(\underline{\lambda})$ be the least value of $L(\cdot, \underline{\lambda})$. Assuming only that all functions are continuous, it is proved that $\phi(\underline{\lambda})$, $\underline{\lambda} \in \mathcal{R}^m$, is concave. Further, if the minimizer of $L(\cdot, \underline{\lambda})$ is unique, then $\phi(\underline{\lambda})$ is differentiable at $\underline{\lambda} \in \mathcal{R}^m$, the components of $\underline{\nabla}\phi(\underline{\lambda})$ being values of the constraint functions. These properties, and some difficulties that occur when the minimizer of $L(\underline{x}, \underline{\lambda})$, $\underline{x} \in \mathcal{X}$, is not unique, are illustrated by an example that has two variables and one equality constraint.

The name SAO stands for Sequential Approximate Optimization. Now quadratic approximations are made to the objective and constraint functions of the given calculation, and the method above is applied using these approximations instead of the original functions, the approximations being updated in an outermost iteration. They have diagonal second derivative matrices, in order that minimizing every $L(\cdot, \underline{\lambda})$ is easy, which allows n to be huge. The quadratic constraints are often inconsistent, however, so the bounds $\|\underline{\lambda}\|_\infty \le \Lambda$ may be imposed for some constant Λ. It is proved that, if $\underline{\lambda}$ maximizes $\phi(\underline{\lambda})$ subject to $\|\underline{\lambda}\|_\infty \le \Lambda$, and if a unique vector $\underline{x}(\underline{\lambda}) \in \mathcal{X}$ minimizes $L(\cdot, \underline{\lambda})$, then $\underline{x}(\underline{\lambda})$ minimizes the objective function plus Λ times the L_1 norm of the violations of the current constraints. This result is highly useful for controlling the updating of the quadratic approximations in the outermost iteration of the SAO method.

Light refreshments will be provided at Room B6605 before the colloquium from 4:00 pm to 4:30 pm. Please come and join us!

**** All interested are welcome ****

For enquiry: 3442-9816

LIU BIE JU CENTRE FOR MATHEMATICAL SCIENCES

香港城市大學
City University of Hong Kong