





# Applied Mathematics 2024

28 May - 1 Jun 2024 City University of Hong Kong



# **International Conference on Applied Mathematics 2024**

28 May – 1 June 2024, City University of Hong Kong

Program and Abstracts

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## Welcome Message

Welcome to City University of Hong Kong (CityU), the host of the International Conference on Applied Mathematics 2024 (ICAM 2024).

The objectives of this series of International Conferences on Applied Mathematics are to review and discuss the latest trends in various fields of applied mathematics. The first ICAM was held in June 2008, and the series continued every two years until 2018. It was disrupted by the COVID pandemic and restarted last year. We believe that the ICAM conference series provides an important platform for exchanging ideas, engaging in in-depth discussions on different aspects of applied mathematics, and sharing expertise, experience, and insights among local, mainland Chinese, and overseas academics.

This year's conference will cover a wide range of topics, including the analysis and computation of waves, approximation theory, sparsity, and deep learning in data science, imaging science, and machine learning, partial differential equations in fluid dynamics, recent advances in inverse problems, scientific computing for partial differential equations, and stochastic analysis.

During the conference, the 2024 William Benter Prize in Applied Mathematics will be awarded. This esteemed prize aims to recognize outstanding mathematical contributions that have had a direct and fundamental impact on scientific, business, finance, or engineering applications.

We are confident that you will enjoy the conference and your stay in Hong Kong, one of the most exciting cities in the Asia-Pacific region.

## **General Information**

### **Organizing Committee**

Raymond CHAN, City University of Hong Kong, Hong Kong
Hongyu LIU, City University of Hong Kong, Hong Kong
Tao LOU, City University of Hong Kong, Hong Kong
Ya Yan LU, City University of Hong Kong, Hong Kong (Chair)
Chenchen MOU, City University of Hong Kong, Hong Kong
Weifeng QIU, City University of Hong Kong, Hong Kong
Jonathan James WYLIE, City University of Hong Kong, Hong Kong
Ding-Xuan ZHOU, The University of Sydney, Australia
Xiaosheng ZHUANG, City University of Hong Kong, Hong Kong

## **Conference Coordinators**

Conference Secretary Nina CHAN Liu Bie Ju Centre for Mathematical Sciences, City University of Hong Kong

Website: https://www.cityu.edu.hk/rcms/icam2024/index.html

#### **REGISTRATION**

Date: 28 May to 30 May (Tuesday to Thursday)
Time: 9:00 am - 9:45 am (Tuesday, 28 May)
8:15 am - 9:15 am (Wednesday, 29 May)
8:00 am - 9:15 am (Thursday, 30 May)

#### **CONFERENCE VENUE**

Wong To Yick Tong Lecture Theatre (LT-17), 4/F, Yeung Kin Man Academic Building, CityU

LT-17 is equipped with a desktop computer, a cable for connecting to laptop, an overhead projector and white boards.

## **SOCIAL EVENTS**

## **Group Photo**

Date: 28 May 2024 (Tue)

Time: 9:30 am

Venue: Wong To Yick Tong Lecture Theatre (LT-17)

## **Conference Banquet** (by Invitation Only)

Date: 30 May 2024 (Thu) Time: 6:30 to 9:30 pm Venue: Faculty Lounge

9/F, Bank of China (Hong Kong) Complex, City University of Hong Kong

HK\$600 for extra banquet ticket

## **Other Information**

#### Name Badges

All attendees are requested to wear their name badge. Conference Secretary and Conference Assistants are ready to assist you if needed.

#### **Banking Service**

Opening hours: 09:00–17:00 (Monday–Friday)

Location: 3/F Yeung Kin Man Academic Building (next to Run Run Shaw Library)



Services including foreign currency and traveler's cheque exchange are provided at Hang Seng bank.

#### **Message Board**

Message boards are located outside Lecture Theatre 17 (LT-17). The latest information of the conference and messages for attendees will be posted on these boards.

#### **Computer & Internet Services**

Networked computers are available at Mathematical Laboratory during the conference period:

Date: 28 May to 1 June 2024 Time: 09:00-12:30, 13:45-17:30

Venue: Y6504, 6/F, Yeung Kin Man Academic Building (Yellow Zone near Lift 9)

Please contact our colleague to unlock the door of the computer lab for you.

Wireless internet access through your own mobile device within CityU campus is also available.

Login name and password can be found at the inner side of your name badge.

# **Dining**

Several canteens are available at the campus and over 30 restaurants can be found at the adjacent shopping mall Festival Walk.

### **CityU Campus**

#### **City Express**



Location: 5/F, Yeung Kin Man Academic Building Opening hours: 07:30–21:00 (Monday–Sunday)

Menu: Fast food, dim sum, short orders, set meal, kebab, vegetarian dish, daily carving,

drinks and bakery products.

#### **AC2 Canteen**



Location: 3/F, Li Dak Sum Yip Yio Chin Academic Building

Opening hours: 07:30–21:00 (Monday–Sunday)

Menu: Fast food

### **City Chinese Restaurant**



Location: 8/F, Bank of China (Hong Kong) Complex

Opening hours: 11:00–15:50 & 17:30-21:30 (Monday–Friday)

09:00–15:50 & 17:30-21:30 (Saturday, Sunday and public holidays)

Menu: Chinese menu with full selection

### **Faculty Lounge**



Location: 9/F, Bank of China (Hong Kong) Complex Opening hours: 11:00–22:30 (Monday–Sunday)

Menu: Western menu

### **AC3 Bistro**



Location: 7/F, Lau Ming Wai Academic Building Opening hours: 07:30–21:00 (Monday–Saturday) Closed on Sunday and public holidays

Menu: Western food

## AC3 Café



Location: 3/F, Lau Ming Wai Academic Building Opening hours: 07:30–21:00 (Monday–Friday) 09:00–19:00 (Saturday and Sunday)

Closed on public holidays

Menu: Sandwich, salad, snacks and drinks

## **Lodge Bistro**



Location: G/F, Academic Exchange Building Opening hours: 07:30–20:30 (Monday–Sunday)

Menu: Western food

## 5380 Cafe (Kebab Station)



Location: 5/F, Bank of China (Hong Kong) Complex Opening hours: 10:00–20:00 (Monday–Saturday) Closed on Sunday and public holidays

Menu: Hot halal food and kebab

## Coffee Cart

Location: Purple Zone, 4/F, Yeung Kin Man Academic Building

Opening hours: 08:00–20:00 (Monday–Friday)

08:00–17:00 (Saturday)

Closed on Sundays and public holidays

Service: Snacks, drinks and stationeries

### **Homey Kitchen**

Location: Student Residence Multi-function Hall B

Opening hours: 9:30 – 15:15 & 15:45 – 19:30 (Monday – Friday)

11:00 – 16:00 (Saturday)

Closed on Sundays and public holidays

Service: Snacks, drinks and stationeries

#### **Festival Walk (Shopping Mall)**

With an extensive selection of restaurants and menus, this adjacent shopping mall provides more choices for dinning. The dinning and shopping directories of the mall can be found in the conference package.

# **Useful Telephone Numbers**

## **CityU Campus**

LBJ Centre: +852 3442 6570 Health Centre: +852 3442 6066 Security Office: +852 3442 8888

## **Hotels**

 Harbour Plaza Metropolis:
 +852 3160 6807

 Eaton, Hong Kong:
 +852 2710 1828

 Royal Park Hotel:
 +852 2695 9291

 InterContinental Grand Stanford Hong Kong:
 +852 2731 2882

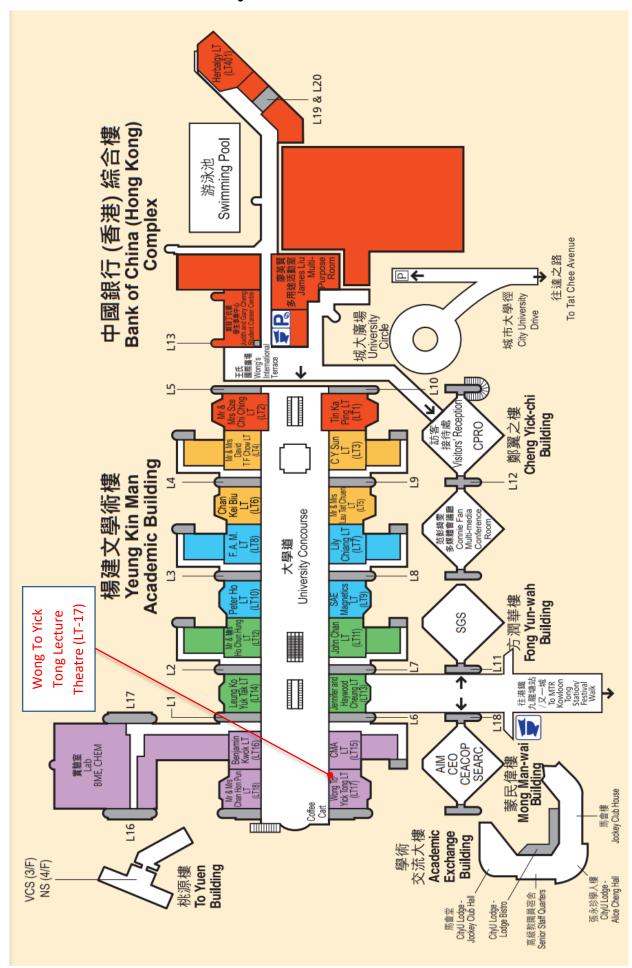
 Royal Plaza Hotel:
 +852 2622 6290

## **MISC**

Immigration Department: +852 2824 6111

Hong Kong Police: 999

# **CityU Location Plan**



<u>Time</u>		28 May 2024, Tuesday			
9:00 - 9:45	Registration				
9:45 – 10:45		LT-17			
		Opening Ceremony			
	Introduction of the Prize Winner - Thomas Yizhao HOU				
		Prize Presentation Ceremony			
		Photo Session			
10:45 – 11:15		Coffee Break			
		LT-17			
		(Chair: Raymond CHAN)			
11:15 – 12: 15	P	LENARY I: Thomas Yizhao HC	<b>D</b> U		
12:15 – 13:45		Lunch Break			
	LT-17				
	(Chair: Jean-Michel MOREL)				
13:45 – 14:45	PLENARY II: Jon KEATING				
14:45 – 15:00	Brief Break				
	LT-17	P4703	P4704		
	(Chair: Yalchin EFENDIEV)	(Chair: Raymond CHAN)	(Chair: Chenchen MOU)		
15:00 – 15:30	A1: Congming LI	B1: Jianfeng CAI	C1: Ludovic TANGPI		
15:30 – 16:00	A2: Zhen LEI	B2: Youwei WEN	C2: Alekos CECCHIN		
16:00 – 16:30		Coffee Break			
16:30 - 17:00	A3: Tao LUO	B3: Hui JI	C3: Zhenjie REN		
17:00 – 17:30	A4: Shu WANG	B4: Ke YIN	C4: Ben SEEGER		
17:30 – 18:00	A5: Lei ZHANG	B5: Ying-Cong CHEN	C5: Lane Chun YEUNG		
	17, Level 4, Yeung Kin Man Acade d P4704: Classrooms, Purple Zone	emic Building (Academic 1) e, Level 4, Yeung Kin Man Academi	c Building (Academic 1)		
		., ,			

A: Special Session for Prof. Thomas HOU

B: Imaging Science and Machine Learning

C: Stochastic Analysis

<u>Time</u> 8:15 - 9:15			<u>29</u>	May 2024, Wednesday Registration		
	LT-17					
				(Chair: Ya Yan LU)		
9:15 - 10:15		P	LENA	ARY III: Thanasis FOI	KAS	
10:15 – 10:45				Coffee Break		
	LT-17			P4703	P4	704
	(Chair: Wing Tat	LEUNG)	(	Chair: Han FENG)	(Chair: Lau	rent MERTZ)
10:45 - 11:15	A6: Yunqing I	IUANG		D1: Bin HAN	C6: Jie	XIONG
11:15 - 11:45	A7: Juan GA	LVIS	Г	2: Ozgur YILMAZ	C7: Vahagn	NERSESYAN
11:45 – 12:15	A8: Maria VAS	ILYEVA	Γ	3: Say Song GOH	C8: Elie	AIDEKON
12:15 - 13:45				Lunch Break		
				LT-17		
			((	Chair: Laurent MERTZ)		
13:45 – 14:45		P	LENA	RY IV: Josselin GARN	IER	
14:45 – 15:00				Brief Break		
	LT-17	P4703		P4704	P4	701
	(Chair: Ming-	(Chair: Ya Yan		(Chair: Jean-Michel	(Chair: Che	nchen MOU)
	Chih LAI)	LU)		MOREL)		
	A9: Zuoqiang	E1: Guang	hui	B6: Chong CHEN	15:00 - 15:20	I1: Kangning
15:00 – 15:30	SHI	HU				CUI
	A10: Min	E2: Yi Zh	łU	B7: Lingfeng LI	15:20 - 15:40	I2: Yuqing LIU
	WANG				15:40 - 16:00	I3: Nathanael
15:30 – 16:00						TEPAKBONG
16:00 - 16:30				Coffee Break		
16:30 - 17:00	A11: Qiuqi LI	E3: Jun L	AI	B8: Ruigang ZHENG	16:30 - 16:50	I4: Buzheng
						SHAN
17:00 – 17:30	A12: Chia-Chieh	E4: Carlo		B9: Ruoning LI	16:50 - 17:10	I5: Wei XIE
	Jay CHU	PÉREZ				
		ARANCIE				
17:30 – 18:00	A13: Wing Tat	E5: Tao Y	IN	B10: Yutong WU		
	LEUNG					
	17, Level 4, Yeung Ki d P4704: Classrooms			ilding (Academic 1) 4, Yeung Kin Man Acade	mic Building (Aca	demic 1)
, ,					1 Cassian for Draf	

A: Special Session for Prof. Thomas HOU
B: Imaging Science and Machine Learning
C: Stochastic Analysis
D: Approximation Theory, Sparsity, and Deep
Learning in Data Science
E: Analysis and Computation of Waves
I: Contributed Talks

<u>Time</u>			<u>30</u> ]	May 2024, Thursday			
8:15 - 9:15	Registration						
	LT-17						
0.15 10.15			`	Chair: Weifeng QIU)			
9:15 - 10:15			PLE	CNARY V: Jie SHEN			
10:15 – 10:45				Coffee Break			
	LT-17		(61	P4703		704	
	(Chair: Wing Ta		·	nair: Wangtao LU)		ongyu LIU)	
10:45 - 11:15	A14: Ming-C		E6:	Weiying ZHENG		ian DIAO	
11:15 - 11:45	A15: Quanling			E7: Peijun LI	F2: Rongt	ang GONG	
11:45 – 12:15	A16: Guang	lian LI	1	E8: Xue JIANG	F3: Yul	kun GUO	
12:15 - 13:45				Lunch Break			
				LT-17			
	(Chair: Xiaosheng ZHUANG)						
13:45 – 14:45		P	LENAF	RY VI: Ding-Xuan ZI	HOU		
	LT-17	P4703	3	P4704	P4	701	
	(Chair: Tao	(Chair: Xiao	(Chair: Xiaosheng (Chair: Bangti Jl		(Chair: Wing	(Chair: Wing Tat LEUNG)	
	LUO)	ZHUANG)					
	G1: Guilong	D4: Xin C	GUO	F4: Haigang LI	15:00 - 15:20	I6: Dmitry	
15:00 – 15:30	GUI					AMMOSOV	
	G2: Jing WANG	D5: Lei S	SHI	F5: Keji LIU	15:20 - 15:40	I7: Zuodong	
						WANG	
15:30 – 16:00					15:40 - 16:00	I8: Jingyu LIU	
16:00 - 16:30				Coffee Danak			
10:00 - 10:50	I T 15	1		Coffee Break		24704	
		P4703 P4700					
17.20 17.00	(Chair: Tao		·	: Xiaosheng ZHUANG		Bangti JIN)	
16:30 - 17:00	G3: Qin D		D	6: Zheng-Chu GUO		aodong LIU	
17:00 – 17:30	G4: Yu M			D7: Yi SHEN	•	MUKHERJEE	
17:30 – 18:00	G5: Jitao		. =	D8: Jun FAN	F8: Hai	bing WANG	
	.7, Level 4, Yeung Ki d P4704: Classrooms			ling (Academic 1) . Yeung Kin Man Acade	mic Building (Aca	demic 1)	

A: Special Session for Prof. Thomas HOU
D: Approximation Theory, Sparsity, and Deep
Learning in Data Science
E: Analysis and Computation of Waves
F: Recent Advances on Inverse Problems
G: Partial Differential Equations in Fluid
Dynamics
I: Contributed Talks

<u>Time</u>	31 May 2024, Friday			
	LT-17			
	(Chair: Hongyu LIU)			
9:15 - 10:15	I	PLENARY VII: Hongkai ZHAO	)	
10:15 – 10:45		Coffee Break		
	LT-17	P4703	P4704	
	(Chair: Shun ZHANG)	(Chair: Ding-Xuan ZHOU)	(Chair: Laurent MERTZ)	
10:45 - 11:15	H1: Lorenzo MASCOTTO	D9: Chenglong BAO	C9: Nan CHEN	
11:15 - 11:45	H2: Zhaonan DONG	D10: Cheng CHENG	C10: Toshihiro YAMADA	
11:45 – 12:15	H3: Yating WANG	D11: Shaobo LIN	C11: Nicolas PRIVAULT	
12:15 - 13:45	Lunch Break			
	LT-17			
	(Chair: Shun ZHANG)			
13:45 – 14:45	PLENARY VIII: Yalchin EFENDIEV			
14:45 – 15:00		Brief Break		
	LT-17	P4703	P4704	
	(Chair: Yanjin WANG)	(Chair: Ya Yan LU)	(Chair: Panpan REN)	
15:00 – 15:30	G6: Beixiang FANG	E9: Li-Lian WANG	C12: Eva KOPFER	
15:30 – 16:00	G7: Bingbing DING	E10: Wei YANG	C13: Michael MCAULEY	
16:00 - 16:30		Coffee Break		
16:30 - 17:00	G8: Tian-Yi WANG	E11: Jiayu QIU	C14: Jing WU	
17:00 – 17:30	G9: Teng WANG	E12: Wangtao LU	C15: Kohei SUZUKI	
17:30 – 18:00	G10: Jie KUANG	E13: Nan ZHANG	C16: Jianhai BAO	
	7, Level 4, Yeung Kin Man Acaden		Duilding (Academic 1)	
P4701, P4703 and P4704: Classrooms, Purple Zone, Level 4, Yeung Kin Man Academic Building (Academic 1)				

C: Stochastic Analysis
D: Approximation Theory, Sparsity, and Deep
Learning in Data Science
E: Analysis and Computation of Waves
G: Partial Differential Equations in Fluid
Dynamics
H: Scientific Computing for Partial Differential
Equations

<u>Time</u>	<u>1 June 2024, Saturday</u>			
	LT-17			
		(Chair: Jonathan WYLIE)		
9:30 - 10:30	]	PLENARY IX: Jon CHAPMAN	I	
10:30 - 11:00		Coffee Break		
	LT-17	P4703	P4704	
	(Chair: Lina ZHAO)	(Chair: Xiaosheng ZHUANG)	(Chair: Bangti JIN)	
11:00 - 11:30	H4: Jian HUANG	D12: Yan-ran LI	F9: Lu CHEN	
11:30 - 12:00	H5: Siyang WANG	D13: Qia LI	F10: Youjun DENG	
12:00 - 12:30	H6: Shubin FU	D14: Yunwen LEI	F11: Yu GAO	
12:30 - 14:00		Lunch Break		
	LT-17	P4703	P4704	
	(Chair: Weifeng QIU)	(Chair: Wei XIANG)	(Chair: Hongyu LIU)	
14:00 – 14:30	H7: Zhongjian WANG	G11: Yanjin WANG	F12: Yan JIANG	
14:30 – 15:00	H8: Zhiwen ZHANG	G12: Tao WANG	F13: Bangti JIN	
15:00 - 15:30		Coffee Break		
15:30 - 16:00	H9: Lina ZHAO	G13: Xumin GU	F14: Hongjie LI	
16:00 – 16:30	H10: Catherine LO	G14: Jiaqi YANG	F15: Guanghui ZHENG	
16:30 – 17:00	H11: Minghui DING	G15: Wenjian PENG	F16: Zhi ZHOU	
	LT-17: Lecture 17, Level 4, Yeung Kin Man Academic Building (Academic 1) P4701, P4703 and P4704: Classrooms, Purple Zone, Level 4, Yeung Kin Man Academic Building (Academic 1)			

D: Approximation Theory, Sparsity, and Deep
Learning in Data Science
F: Recent Advances on Inverse Problems
G: Partial Differential Equations in Fluid
Dynamics
H: Scientific Computing for Partial Differential
Equations

ICAM 2024, Day 1, 28 May 2024, Tuesday

HK Time	10/11/1 2024, Day 1, 20 May 2024, Incomy	Venue
9:00 - 9:45	Registration opens	LT-17
9:45 - 10:45	Opening Ceremony	LT-17
	Introduction of 2024 William Benter Prize Winner - Prof. Thomas Yizhao HOU	
	Prof. Yalchin EFENDIEV, Texas A & M University, USA	
	Prize Presentation Ceremony	
	Photo Session	
10:45 - 11:15	Coffee Break	
10:45 - 11:45	PLENARY TALK I: Potentially singular behavior of 3D incompressible Navier-	LT-17
	Stokes equations	
	Prof. Thomas Yizhao HOU, Caltech, USA	
	Session Chair: Prof. Raymond CHAN	
12:15 - 13:15	Lunch Break	
13:45 - 15:00	PLENARY TALK II: Machine learning and random matrices	
	Prof. Jon KEATING, University of Oxford, UK	
	Session Chair: Prof. Jean-Michel MOREL	
14:45 – 15:00	Brief Break	
	PARALLEL SESSION	
	Special Session for Prof. Thomas HOU (Session Chair: Prof. Yalchin EFENDIEV)	LT-17
15:00 – 15:30	Some work on the analysis of equations of fluid type	
	Congming LI, Shanghai Jiao Tong University and CMA of Shanghai, China	
15:30 – 16:00	Energy cascade of Klein-Gordon	
	Zhen LEI, Fudan University, China	
	Imaging Science and Machine Learning (Session Chair: Prof. Raymond CHAN)	P4703
15:00 – 15:30	Interlacing polynomial method for matrix approximation	
	Jianfeng CAI, The Hong Kong University of Science and Technology, Hong Kong	
15:30 – 16:00	Image segmentation using Bayesian inference for convex variant Mumford-Shah variational model	
	Youwei WEN, Hunan Normal University, China	
	Stochastic Analysis (Session Chair: Prof. Chenchen MOU)	P4704
15:00 – 15:30	Mean field games in discrete time	
	Ludovic TANGPI, Princeton University, USA	
15:30 – 16:00	On the long time behavior of mean field control problems	
	Alekos CECCHIN, University of Padova, Italy	

16:00 – 16:30	Coffee Break	
	PARALLEL SESSION	
	Special Session for Prof. Thomas HOU (Session Chair: Prof. Yalchin EFENDIEV)	LT-17
16:30 - 17:00	On a free boundary problem of 3-D compressible Euler equations coupled with a nonlinear Poisson equation	
	Tao LUO, City University of Hong Kong, Hong Kong	
17:00 - 17:30	Quasi-neutral limit and the boundary layer problem of Planck-Nernst-Poisson-Navier-Stokes equations for electrohydrodynamics	
	Shu WANG, Beijing University of Technology, China	
17:30 - 18:00	Operator learning for multiscale PDEs	
	Lei ZHANG, Shanghai Jiao Tong University, China	
	Imaging Science and Machine Learning (Session Chair: Prof. Raymond CHAN)	P4703
16:30 - 17:00	Neural expectation maximization for self-supervised blind image deblurring	
	Hui JI, National University of Singapore, Singapore	
17:00 - 17:30	A smoothing technique for non-smooth optimization with applications to constrained minimax problems	
	Ke YIN, Huazhong University of Science and Technology, China	
17:30 - 18:00	Towards high-fidelity text-to-3D generation	
	Ying-Cong CHEN, Hong Kong University of Science and Technology (Guangzhou), China	
	Stochastic Analysis (Session Chair: Prof. Chenchen MOU)	P4704
16:30 - 17:00	Regularized mean field optimization with application to neural networks	
	Zhenjie REN, University Paris Dauphine, PSL, France	
17:00 - 17:30	Equations on Wasserstein space and applications	
	Ben SEEGER, University of North Carolina at Chapel Hill, USA	
17:30 - 18:00	Rank-based models with common noise and pathwise entropy solutions of SPDEs	
	Lane Chun YEUNG, Carnegie Mellon University, USA	

ICAM 2024, Day 2, 29 May 2024, Wednesday

HK Time	TCAM 2024, Day 2, 2) May 2024, Wednesday	Venue
8:15 - 9:15	Registration opens	LT-17
9:15 - 10:15	PLENARY TALK III: The beauty and power of the complex plane	LT-17
	Prof. Thanasis FOKAS, University of Cambridge, UK	
	Session Chair: Prof. Ya Yan LU	
10:15 - 10:45	Coffee Break	-
	PARALLEL SESSION	
	Special Session for Prof. Thomas HOU (Session Chair: Prof. Wing Tat LEUNG)	LT-17
10:45 - 11:15	On the optimal order approximation of the partition of unity finite element method	
	Yunqing HUANG, Xiangtan University, China	
11:15 - 11:45	FEM	
	Juan GALVIS, Universidad Nacional de Colombia, Colombia	
11:45 - 12:15	Multiscale model order reduction for network models	
	Maria VASILYEVA, Texas A&M University – Corpus Christi, USA	
	<b>Approximation Theory, Sparsity, and Deep Learning in Data Science</b> (Session Chair: Prof. Han FENG)	P4703
0:45 - 11:15	Wavelet methods for the elliptic interface problems	
	Bin HAN, University of Alberta, Canada	
11:15 - 11:45	Generative compressed sensing with Fourier measurements	
	Ozgur YILMAZ, The University of British Columbia, Canada	
11:45 - 12:15	Explicit constructions of pairs of dual frames on locally compact abelian groups	
	Say Song GOH, National University of Singapore, Singapore	
	Stochastic Analysis (Session Chair: Prof. Laurent MERTZ)	P4704
10:45 - 11:15	Stochastic maximum principle for weighted mean-field system	
	Jie XIONG, Southern University of Science and Technology, Shenzhen, China	
11:15 - 11:45	Controllability and stochastic PDEs	
	Vahagn NERSESYAN, NYU Shanghai, China	
11:45 - 12:15	A connection between skew Brownian motion and BESQ flow	
	Elie AIDEKON, Fudan University, China	
12:15 - 13:45	Lunch Break	
13:45 - 14:45	PLENARY TALK IV: Reduced order model approach for imaging with waves	LT-17
	Prof. Josselin GARNIER, Ecole polytechnique, France	
	Session Chair: Prof. Laurent MERTZ	
14:45 - 15:00	Brief Break	

	PARALLEL SESSION	
	Special Session for Prof. Thomas HOU (Session Chair: Prof. Ming-Chih LAI)	LT-17
15:00 - 15:30	Convection-diffusion equation: An axiomatized framework for neural networks	
	Zuoqiang SHI, Tsinghua University, China	
15:30 - 16:00	The interplay between deep learning and model reduction	
	Min WANG, University of Houston, USA	
	Analysis and Computation of Waves (Session Chair: Prof. Ya Yan LU)	P4703
15:00 - 15:30	Surface wave scattering problems in a periodic inhomogeneous layer with local defects	
	Guanghui HU, Nankai University, China	
15:30 - 16:00	Topologically protected waves in superhoneycomb structures	
	Yi ZHU, Tsinghua University, China	
	Imaging Science and Machine Learning (Session Chair: Prof. Jean-Michel MOREL)	P4704
15:00 - 15:30	Convergence analysis of nonlinear Kaczmarz method for systems of nonlinear equations with component-wise convex mapping	
	Chong CHEN, Chinese Academy of Sciences, China	
15:30 - 16:00	A priori error estimate of deep mixed residual method for elliptic PDEs	
	Lingfeng LI, Hong Kong Center for Cerebro-Cardiovascular Health Engineering, Hong Kong	
	Contributed Talks (Session Chair: Prof. Chenchen MOU)	P4701
15:00 - 15:20	Superpixel-based and spatially-regularized diffusion learning for unsupervised hyperspectral image clustering	
	Kangning CUI, City University of Hong Kong, Hong Kong	
15:20 - 15:40	Approximation of functions from Korobov spaces by shallow RELU neural networks	
	Yuqing LIU, City University of Hong Kong, Hong Kong	
15:40 - 16:00	Fast asymptotic rates of convergence for neural networks under the hard margin condition  Nathanael TEPAKBONG, City University of Hong Kong, Hong Kong	
16:00 - 16:30	Coffee Break	_
	Special Session for Prof. Thomas HOU (Session Chair: Prof. Ming-Chih LAI)	LT-17
16:30 - 17:00	Model reduction methods based on DMD for parametric dynamical systems	
	Qiuqi LI, Hunan University, China	
17:00 - 17:30	Equivalent extensions of partial differential equations on curves or surfaces	
	Chia-Chieh Jay CHU, National Tsing Hua University, Taiwan	
17:30 - 18:00	Multicontinuum homogenization and its application	
	Wing Tat LEUNG, City University of Hong Kong, Hong Kong	

	Analysis and Computation of Waves (Session Chair: Prof. Ya Yan LU)	P4703
16:30 - 17:00	The singularity swapping method for boundary integral equations and its applications to wave scattering	
	Jun LAI, Zhejiang University, Zhejiang, China	
17:00 - 17:30	Density interpolation methods for boundary/volume integral equations in wave phenomena	
	Carlos PÉREZ-ARANCIBIA, University of Twente, The Netherlands	
17:30 - 18:00	Boundary integral equation solvers for layered-medium scattering problems	
	Tao YIN, Chinese Academy of Sciences, China	
	Imaging Science and Machine Learning (Session Chair: Prof. Jean-Michel MOREL)	P4704
16:30 - 17:00	Data-adaptive graph framelets with generalized vanishing moments for graph signal processing	
	Ruigang ZHENG, City University of Hong Kong, Hong Kong	
17:00 - 17:30	Spectral-Spatial Classification Methods for Hyperspectral Images	
	Ruoning LI, City University of Hong Kong, Hong Kong	
17:30 - 18:00	A phase tomography model in adaptive optics system	
	Yutong WU, City University of Hong Kong, Hong Kong	
	Contributed Talks (Session Chair: Prof. Chenchen MOU)	P4701
16:30 - 16:50	Multicontinuum Homogenization for Coupled Flow and Transport Equations	
	Buzheng SHAN, Texas A&M University, USA	
16:50 - 17:10	Multicontinuum homogenization in perforated domains	
	Wei XIE, Xiangtan University, China	

ICAM 2024, Day 3, 30 May 2024, Thursday

HK Time		Venue
8:15 - 9:15	Registration opens	LT-17
9:15 - 10:15	PLENARY TALK V: Recent advances on stiffly stable higher-order IMEX schemes with applications to incompressible Navier-Stokes equations	LT-17
	Prof. Jie SHEN, Eastern Institute of Technology, China	
	Session Chair: Prof. Weifeng QIU	
10:15 – 10:45	Coffee Break	
	PARALLEL SESSION	
	Special Session for Prof. Thomas HOU (Session Chair: Prof. Wing Tat LEUNG)	LT-17
10:45 - 11:15	A feature-capturing PINN for Stokes problems with discontinuous viscosity and singular forces	
	Ming-Chih LAI, National Yang Ming Chiao Tung University, Taiwan	
11:15 - 11:45	Sharper guaranteed lower bounds for eigenvalues	
	Quanling DENG, Australian National University, Australia	
11:45 - 12:15	Wavelet-based edge multiscale finite element methods for singularly perturbed convection-diffusion equations	
	Guanglian LI, The University of Hong Kong, Hong Kong	
	Analysis and Computation of Waves (Session Chair: Prof. Wangtao LU)	P4703
10:45 - 11:15	Perfectly matched layer method for the wave scattering problem by a step-like surface	
	Weiying ZHENG, Chinese Academy of Sciences, China	
11:15 - 11:45	A novel boundary integral formulation for the biharmonic wave scattering problem	
	Peijun LI, Academy of Sciences, China	
11:45 - 12:15	A PML method for signal-propagation problems in axon	
	Xue JIANG, Beijing University of Technology, China	
	Recent Advances on Inverse Problems (Session Chair: Prof. Hongyu LIU)	P4704
10:45 - 11:15	Visibility, invisibility and unique recovery of inverse electromagnetic problems with conical singularities	
	Huaian DIAO, Jilin University, China	
11:15 - 11:45	The coupled complex boundary methods for inverse problems of partial differential equations	
	Rongfang GONG, Nanjing University of Aeronautics and Astronautics, China	
11:45 - 12:15	Towards a novel iterative method for the inverse elastic scattering problem	
	Yukun GUO, Harbin Institute of Technology, China	

12:15 - 13:45	Lunch Break	
13:45 – 14:45	PLENARY TALK VI: Mathematical theory of structured deep neural networks	LT-17
	Prof. Ding-Xuan ZHOU, University of Sydney, Australia	
	Session Chair: Prof. Xiaosheng ZHUANG	
	PARALLEL SESSION	
	Partial Differential Equations in Fluid Dynamics (Session Chair: Prof. Tao LUO)	LT-17
15:00 - 15:30	Vanishing surface tension limit of viscous surface wave equations in an infinite layer	
	Guilong GUI, Xiangtan University, China	
15:30 - 16:00	Uniform regularity and vanishing dissipation limit for the 3D magnetic B´enard equations in half space	
	Jing WANG, Shanghai Normal University, China	
	<b>Approximation Theory, Sparsity, and Deep Learning in Data Science</b> (Session Chair: Prof. Xiaosheng ZHUANG)	P4703
15:00 - 15:30	Centered kernels and pairwise learning	
	Xin GUO, The University of Queensland, Australia	
15:30 - 16:00	Classification with deep neural networks	
	Lei SHI, Fudan University, China	
	Recent Advances on Inverse Problems (Session Chair: Prof. Bangti JIN)	P4704
15:00 - 15:30	Babu ska problem in composite materials: PDE analysis and FEM	
	Haigang LI, Beijing Normal University, China	
15:30 - 16:00	The inverse problems for a DCIS model with free boundaries in mathematical biology	
	Keji LIU, Shanghai University of Finance and Economics, China	
	Contributed Talks (Session Chair: Prof. Wing Tat LEUNG)	P4701
15:00 - 15:20	Multiscale model reduction for a nonlinear strain-limiting Cosserat elasticity model using GMsFEM	
	Dmitry AMMOSOV, North-Eastern Federal University, Yakutsk, Russia	
15:20 - 15:40	Asymptotic and invariant-domain preserving schemes for scalar conservation equations with stiff source terms and multiple equilibrium points	
	Zuodong WANG, Centre Inria de Paris & Ecole des ponts ParisTech, France	
15:40 - 16:00	Analysis of the staggered DG method for the quasi-Newtonian Stokes flows	
	Jingyu LIU, City University of Hong Kong, Hong Kong	
16:00 - 16:30	Coffee Break	
	Partial Differential Equations in Fluid Dynamics (Session Chair: Prof. Tao LUO)	LT-17
16:30 - 17:00	Well-posedness of regular solutions for 3D full compressible Navier-Stokes equations with degenerated viscosities	
	Qin DUAN, Shenzhen University, China	

	On free boundary problems of compressible viscoelastic fluid with or without surface tension	
17:30 - 18:00	Yu MEI, Polytechnical University, China	
	Asymptotic stability for n-dimensional magnetohydrodynamic equations	
	Jitao LIU, Beijing University of Technology, China	
	<b>Approximation Theory, Sparsity, and Deep Learning in Data Science</b> (Session Chair: Prof. Xiaosheng ZHUANG)	
16:30 - 17:00	Learning theory of spectral algorithms under covariate shift	P4703
	Zheng-Chu GUO, Zhejiang University, Hangzhou, China	1 .700
17:00 - 17:30	Stability of the Frank Wolfe algorithm for compressible signals	
	Yi SHEN, Zhejiang Sci-Tech University, China	
17:30 - 18:00	Information theoretic learning meets deep neural networks	
	Jun FAN, Hong Kong Baptist University, Hong Kong	
	Recent Advances on Inverse Problems (Session Chair: Prof. Bangti JIN)	P4704
16:30 - 17:00	Inverse scattering with multifrequency far field patterns at sparse observation directions	
	Xiaodong LIU, Chinese Academy of Sciences, China	
17:00 - 17:30	Mathematical analysis of therapy modalities using acoustic cavitation	
	Arpan MUKHERJEE, Austrian Academy of Sciences, Austria	
17:30 - 18:00	Inverse problems for diffusion equations	
	Haibing WANG, Southeast University, China	

# ICAM 2024, Day 4, 31 May 2024, Friday

<b>HK Time</b> 9:15 - 10:15	PLENARY TALK VII: Numerical understanding of neural networks: from representation	Venue LT-17
,,,,	to learning dynamics	
	Prof. Hongkai ZHAO, Duke University, USA	
	Session Chair: Prof. Hongyu LIU	
10:15 – 10:45	Coffee Break	
10.12 10.12	PARALLEL SESSION	
	Scientific Computing for Partial Differential Equations (Session Chair: Prof. Shun	LT-17
	ZHANG)	21 1,
10:45 - 11:15	Residual-based a posteriori error estimates for an hpdiscontinuous Galerkin method of the biharmonic problem	
	Lorenzo MASCOTTO, University of Milano-Bicocca, Italy	
11:15 - 11:45	A hypocoercivity-exploiting stabilised finite element method for Kolmogorov equation	
11.15 11.45	Zhaonan DONG, Philip J. Herbert Inria Paris, France	
11:45 - 12:15	Partially explicit splitting scheme with explicit-implicitnull method for nonlinear multiscale flow problems	
	Yating WANG, Xi'an Jiaotong University, China	
	<b>Approximation Theory, Sparsity, and Deep Learning in Data Science</b> (Session Chair: Prof. Ding-Xuan ZHOU)	P4703
10:45 - 11:15	Addressing preferred orientation in single-particle cryo-EM through AI-generated auxiliary particles	
	Chenglong BAO, Tsinghua University, China	
11:15 - 11:45	Svd-based graph Fourier transforms on directed graphs	
	Cheng CHENG, Sun Yat-sen University, China	
11:45 - 12:15	Integral operator approaches for spherical data fitting	
	Shaobo LIN, Xi'an Jiaotong University, China	
	Stochastic Analysis (Session Chair: Prof. Laurent MERTZ)	P4704
10:45 - 11:15	A two time-scale evolutionary game approach to multiagent reinforcement learning and its application in algorithmic collusion	
	Nan CHEN, Chinese University of Hong Kong, Hong Kong	
11:15 - 11:45	Neural network SDE simulator	
	Toshihiro YAMADA, Hitotsubashi University, Japan	
11:45 - 12:15	Probabilistic representation of the solutions of nonlocal partial differential equations	
	Nicolas PRIVAULT, Nanyang Technological University, Singapore	

13:45 - 14:45	PLENARY TALK VIII: Multiscale finite element methods and multicontinuum	T = 15
		LT-17
	homogenization	
	Prof. Yalchin EFENDIEV, Texas A & M University, USA	
	Session Chair: Prof. Shun ZHANG	
14:45 - 15:00	Brief Break	
	PARALLEL SESSION	
	Partial Differential Equations in Fluid Dynamics (Session Chair: Prof. Yanjin WANG)	LT-17
15:00 - 15:30	A new approach to transonic shock solutions for steady 1-D Euler-Poisson equations in an interval	
	Beixiang FANG, Shanghai Jiao Tong University, China	
15:30 - 16:00	Global smooth solutions to the isentropic and irrotational Chaplygin gases with large data	
	Bingbing DING, Nanjing Normal University, China	
	Analysis and Computation of Waves (Session Chair: Prof. Ya Yan LU)	P4703
15:00 - 15:30	Radial PML-type techniques for wave scattering problems: Real versus complex coordinate transformations	
	Li-Lian WANG, Nanyang Technological University, Singapore	
15:30 - 16:00	Time-domain mathematical modeling, finite element simulation, and design in complex anisotropic electromagnetic metamaterials	
	Wei YANG, Xiangtan University, China	
	Stochastic Analysis (Session Chair: Prof. Panpan REN)	P4704
15:00 - 15:30	Homogenisation of discrete dynamical optimal transport	
	Eva KOPFER, University Bonn, Germany	
15:30 - 16:00	Geometric functionals of smooth Gaussian fields	
	Michael MCAULEY, Technological University Dublin, Ireland	
16:00 - 16:30	Coffee Break	
	Partial Differential Equations in Fluid Dynamics (Session Chair: Prof. Yanjin WANG)	LT-17
16:30 - 17:00	On the asymptotic behavior of steady Euler equations	
	Tian-Yi WANG, Wuhan University of Technology, China	
17:00 - 17:30	Asymptotic behavior of solution for the compressible Navier-Stokes equations	
	Teng WANG, Wuhan University, China	
17:30 - 18:00	Hypersonic similarity for steady hypersonic flow over two-dimensional Lipschitz wedge Jie KUANG, Innovation Academy for Precision Measurement Science and Technology, China	
-	Analysis and Computation of Waves (Session Chair: Prof. Ya Yan LU)	
16:30 - 17:00	The bifurcation of Dirac points in photonic/phononic structures	

	Jiayu QIU, Hong Kong University of Science and Technology, Hong Kong	P4703
17:00 - 17:30	A high-accuracy mode solver for acoustic scattering by a periodic array of axially symmetric obstacles	
	Wangtao LU, Zhejiang University, China	
17:30 - 18:00	Computing resonant modes for biperiodic structures	
	Nan ZHANG, City University of Hong Kong, Hong Kong	
	Stochastic Analysis (Session Chair: Prof. Panpan REN)	P4704
16:30 - 17:00	On path and distribution dependent stochastic differential equations with subdifferential equations	
	Jing WU, Sun Yat-sen University, China	
17:00 - 17:30	Dyson Brownian motion as a Wasserstein gradient flow	
	Kohei SUZUKI, Durham University, UK	
17:30 - 18:00	Uniform-in-time estimates for mean-field type SDEs and applications	
	Jianhai BAO, Tianjin University, China	

# ICAM 2024, Day 5, 1 June 2024, Saturday

HK Time 9:30 - 10:30	PLENARY TALK IX: Exponential asymptotics and applied mathematics Jon CHAPMAN, University of Oxford, UK	Venue LT-17
10.20 11.00	Session Chair: Prof. Jonathan WYLIE	
10:30 - 11:00	Coffee Break	
	PARALLEL SESSION	
	Scientific Computing for Partial Differential Equations (Session Chair: Prof. Lina ZHAO)	LT-17
11:00 - 11:30	Characteristic block-centered finite difference methods for Darcy-Forchheimer compressible miscible displacement problem	
	Jian HUANG, Xiangtan University & National Center for Applied Mathematics in Hunan, China	
11:30 - 12:00	A stable and high-order hybrid numerical method for wave propagation problems	
	Siyang WANG, Ume°a University, Sweden	
12:00 - 12:30	Efficient preconditioners for large-scale highly heterogeneous Darcy flow	
	Shubin FU, Eastern Institute of Technology, China	
	<b>Approximation Theory, Sparsity, and Deep Learning in Data Science</b> (Session Chair: Prof. Xiaosheng ZHUANG)	P4703
11:00 - 11:30	Exploring structural sparsity of coil images from 3-dimensional directional tight framelets for SENSE reconstruction	
	Yan-ran LI, Shenzhen University, China	
11:30 - 12:00	Smoothing algorithms for nonsmooth optimization over the Stiefel manifold with applications to the graph Fourier basis problem	
	Qia LI, Sun Yat-sen University, China	
12:00 - 12:30	Generalization and optimization of gradient methods for shallow neural networks	
	Yunwen LEI, The University of Hong Kong, Hong Kong	
	Recent Advances on Inverse Problems (Session Chair: Prof. Bangti JIN)	P4704
11:00 - 11:30	A Scattering theory on hyperbolic spaces	
	Lu Chen, Beijing Institute of Technology, Beijing, China	
11:30 - 12:00	Mathematical framework for multi-layered medium	
	Youjun DENG, Central South University, China	
12:00 - 12:30	Machine learning for inverse scattering problems	
	Yu GAO, Jilin University, China	

12:30 - 14:00	Lunch Break	/
	PARALLEL SESSION	
	Scientific Computing for Partial Differential Equations (Session Chair: Prof. Weifeng QIU)	LT-17
14:00 - 14:30	Solving Inverse Problem by Diffusion Model	
	Zhongjian Wang, Nanyang Technological University, Singapore	
14:30 - 15:00	A novel stochastic interacting particle-field algorithm for solving the 3D parabolic-parabolic Keller-Segel chemotaxis system	
	Zhiwen ZHANG, The University of Hong Kong, Hong Kong	
	Partial Differential Equations in Fluid Dynamics (Session Chair: Prof. Wei XIANG)	P4703
14:00 - 14:30	Global well-posedness of compressible viscous surface waves	
	Yanjin WANG, Xiamen University, China	
14:30 - 15:00	Vacuum free boundary problems in ideal compressible magnetohydrodynamics	
	Tao WANG, Wuhan University, China	
	Recent Advances on Inverse Problems (Session Chair: Prof. Hongyu LIU)	P4704
14:00 - 14:30	Spectral patterns of elastic transmission eigenfunctions: boundary localisation, surface resonance and stress concentration	
	Yan JIANG, Jilin University, China	
14:30 - 15:00	On the early stopping of untrained convolutional neural networks	
	Bangti JIN, The Chinese University of Hong Kong, Hong Kong	
15:00 - 15:30	Coffee Break	
	Scientific Computing for Partial Differential Equations (Session Chair: Prof. Weifeng QIU)	LT-17
15:30 - 16:00	H(div)-conforming HDG methods for the stress-velocity formulation of the Stokes equations and the Navier-Stokes equations	
	Lina ZHAO, City University of Hong Kong, Hong Kong	
16:00 - 16:30	On inverse problems in predator-prey models	
	Catherine LO, City University of Hong Kong, Hong Kong	
16:30 - 17:00	Determining sources in the bioluminescence tomography problem	
	Minghui DING, City University of Hong Kong, Hong Kong	
	Partial Differential Equations in Fluid Dynamics (Session Chair: Prof. Wei XIANG)	
15:30 - 16:00	Local well-posedness for the free boundary incompressible elastodynamics via vanishing viscosity limit	P4703
	Xumin GU, Shanghai University of Finance and Economics, China	11703
16:00 - 16:30	Well-posedness of the free surface problem for non-Newtonian fluids between cylinders rotating at different speeds: <i>Weissenberg effect</i>	
	Jiaqi YANG, Zhejiang University, China	
16:30 - 17:00	The axisymmetric solution of 2-dimensional compressible Euler equations with discontinuous swirl	

	Wenjian PENG, City University of Hong Kong, Hong Kong	
	Recent Advances on Inverse Problems (Session Chair: Prof. Hongyu LIU)	P4704
15:30 - 16:00	The effective construction on elastic metamaterials	
	Hongjie LI, Tsinghua University, China	
16:00 - 16:30	Microscale hydrodynamic cloaking and shielding for electroosmotic model	
	Guanghui ZHENG, Hunan University, Changsha, China	
16:30 - 17:00	Conductivity imaging using deep neural networks	
	Zhi ZHOU, The Hong Kong Polytechnic University, Hong Kong	

# Abstracts

# $\begin{array}{c} \hbox{International Conference on Applied Mathematics} \\ 2024 \end{array}$

28 May - 1 June, 2024 City University of Hong Kong

Co-organized by Liu Bie Ju Centre for Mathematical Sciences Department of Mathematics

### Contents

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### 1 Plenary Talks

#### Exponential asymptotics and applied mathematics

Jon Chapman

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"Divergent series are the invention of the devil, and it is shameful to base on them any demonstration whatsoever." — N. H. Abel.

The lecture will introduce the concept of an asymptotic series, showing how useful divergent series can be, despite Abel's reservations. We will then discuss Stokes' phenomenon, whereby the coefficients in the series appear to change discontinuously. We will show how understanding Stokes' phenomenon is the key which allows us to determine the qualitative and quantitative behaviour of the solution in many practical problems. Examples will be drawn from the areas of surface waves on fluids, crystal growth, dislocation dynamics, Hele-Shaw flow, thin film rupture, quantum mechanics, and atmospheric dynamics.

# Multiscale finite element methods and multicontinuum homogenization

Yalchin Efendiev

Texas A & M University, College Station, TX 77845

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In this talk, I will discuss multucontinuum homogenization and multiscale finite element methods. In multiscale finite element methods, the solution is approximated via a set of multiscale basis functions that are constructed based on local solutions. Multiscale finite element methods are used to solve many practical problems. Multicontinuum homogenization methods use local solution in a manner that gives some type of a continuity of coarse-grid nodal values. As a result, one can use coarse-grid smooth functions as macroscopic variables. The resulting multicontinuum homogenization gives a general framework for solving high contrast problems. We present a general theory and applications.

#### The beauty and power of the complex plane

THANASIS FOKAS

University of Cambridge, UK

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Two important applications of appropriate deformations to the complex plane will be reviewed: the Unified Transform (also known as the Fokas method), and the development of a new methodology for the asymptotic analysis of the Riemann zeta, and related functions.

#### Reduced order model approach for imaging with waves

Josselin Garnier

Centre de Mathématiques Appliquées, Ecole polytechnique, France *Email:* josselin.garnier@polytechnique.edu

We consider the inverse problem for the scalar wave equation. Sensors probe the unknown medium to be imaged with a pulse and measure the backscattered waves. The objective is to estimate the velocity map from the array response matrix of the sensors. Under such circumstances, conventional Full Waveform Inversion (FWI) can be carried out by nonlinear least-squares data fitting. It turns out that the FWI misfit function is high-dimensional and non-convex and it has many local minima. A novel approach to FWI based on a data-driven reduced order model (ROM) of the wave equation operator is introduced and it is shown that the minimization of ROM misfit function performs much better.

The talk is based on a joint work with L. Borcea (Univ. Michigan), A. Mamonov (Univ. Houston), J. Zimmerling (Uppsala Univ.)

# Potentially singular behavior of 3D incompressible Navier-Stokes equations

THOMAS Y. HOU

Department of Computing and Mathematical Sciences, Caltech, USA *Email:* hou@cms.caltech.edu

Whether the 3D incompressible Navier-Stokes equations can develop a finite time singularity from smooth initial data is one of the most challenging problems in nonlinear PDEs. In this talk, I will present some numerical evidence that the 3D Navier-Stokes equations develop nearly self-similar singular scaling properties with maximum vorticity increased by a factor of  $10^7$ . This potentially singular behavior is induced by a potential finite time singularity of the 3D Euler equations. Unlike the Hou-Luo blowup scenario, the potential singularity of the 3D Euler and Navier-Stokes equations occurs at the origin. We have applied several blowup criteria to study the potentially singular behavior of the Navier-Stokes equations. The Beale-Kato-Majda blow-up criterion, the blowup criteria based on the growth of enstrophy and negative pressure, the Ladyzhenskaya-Prodi-Serrin regularity criteria all seem to imply that the Navier-Stokes equations develop potentially singular behavior. Finally, we present some new numerical evidence that a variant of the axisymmetric Navier-Stokes equations with time dependent fractional dimension develops nearly self-similar blowup with maximum vorticity increased by a factor of  $10^{32}$ .

### Machine learning and random matrices

JON KEATING

Mathematical Institute, University of Oxford, UK

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I will discuss some applications of random matrix theory to problems in machine learning.

# Recent advances on stiffly stable higher-order IMEX schemes with applications to incompressible Navier-Stokes equations

JIE SHEN

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How to construct stable second- and higher-order fully decoupled schemes for the incompressible Navier-Stokes equations has been a long standing open problem. A main issue is that stability regions of usual multistep time discretization decrease as their order of accuracy increase, so they do not possess enough stability to control the higher-order explicit treatment of the pressure in a fully decoupled scheme. We shall construct a new class of IMEX schemes, by using Taylor expansion at  $t_{n+\beta}$  (with  $\beta \geq 1$  as a parameter) for updating the solution at  $t_{n+1}$ , whose stability region increases with  $\beta$ , thus allowing us to choose  $\beta$  according to the stability and accuracy requirement. In particular, by choosing suitable  $\beta$ , we are able to construct higher-order unconditionally stable (in  $H^1$  norm), fully decoupled consistent splitting schemes for the Navier-Stokes equations, and derive uniform optimal error estimates.

We shall also present ample numerical results to show the computational advantages of these schemes for some nonlinear parabolic systems, including in particular Navier-Stokes equations.

# Numerical understanding of neural networks: from representation to learning dynamics

HONGKAI ZHAO Duke University, USA

Email: zhao@math.duke.edu

In this talk I will present both analysis and experiments to understand a few basic computational issues in using neural network to approximate functions:

- (1) the numerical error that can be achieved given a finite machine precision,
- (2) the learning dynamics and computation cost to achieve certain accuracy, and (3) structured and balanced approximation. These issues are studied for both approximation and optimization in asymptotic and non-asymptotic regimes.

### Mathematical theory of structured deep neural networks

DING-XUAN ZHOU

School of Mathematics and Statistics, University of Sydney, Australia *Email:* dingxuan.zhou@sydney.edu.au

Deep learning has been widely applied and brought breakthroughs in speech recognition, computer vision, natural language processing, and many other domains. The involved deep neural network architectures and computational issues have been well studied in machine learning. But a theoretical foundation for understanding the modelling, approximation or generalization ability of deep learning models with network architectures is still in progress. An important family of structured deep neural networks is deep convolutional

neural networks (CNNs) with convolutional structures. The convolutional architecture gives essential differences between deep CNNs and fully-connected neural networks, and the classical approximation theory for fully-connected networks developed around 35 years ago does not apply. This talk describes approximation and generalization analysis of deep CNNs and related structured deep neural networks.

### 2 Invited Talks

### A connection between skew Brownian motion and BESQ flow

ELIE AIDEKON\*, CHENGSHI WANG, YAOLIN YU School of Mathematical Sciences, Fudan University, China *Email*: aidekon@fudan.edu.cn

The skew Brownian motion (Itô-McKean, 1965, Walsh, 1978) is a diffusion which behaves as a Brownian motion away from 0, but has a "local drift" every time it crosses 0. It can be constructed by assigning signs to Brownian excursions away from 0, each excursion being positive with probability p and negative with probability 1-p. It can equivalently (Harrison-Shepp, 1981) be seen as the strong solution of the SDE  $dX_t = dB_t + qdL_t(X)$  where  $L_t(X)$  denotes the local time of the diffusion at 0, and q = 2p - 1. We show how this process naturally arises when exploring a flow of squared Bessel processes driven by a white noise.

### Addressing preferred orientation in single-particle cryo-EM through AI-generated auxiliary particles

CHENGLONG BAO

Yau Mathematical Sciences Center, Tsinghua University Beijing Institute of Mathematical Sciences and Applications State Key Laboratory of Membrane Biology, Tsinghua University, China Email: clbao@tsinghua.edu.cn

The single-particle cryo-EM field faces the persistent challenge of preferred orientation, lacking general computational solutions. We introduce cryoPROS, an AI-based approach designed to address the above issue. By generating the auxiliary particles with a conditional deep generative model, cryoPROS addresses the intrinsic bias in orientation estimation for the observed particles. We effectively employed cryoPROS in the cryo-EM single particle analysis of the hemagglutinin trimer, showing the ability to restore the near-atomic resolution structure on non-tilt data. Moreover, the enhanced version named cryoPROS-MP significantly improves the resolution of the membrane protein Na<sub>X</sub> using the no-tilted data that contains the effects of micelles. Compared to the classical approaches, cryoPROS does not need special experimental or

image acquisition techniques, providing a purely computational yet effective solution for the preferred orientation problem. Finally, we conduct extensive experiments that establish the low risk of model bias and the high robustness of cryoPROS.

### Uniform-in-time estimates for mean-field type SDEs and applications

JIANHAI BAO

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Via constructing an asymptotic coupling by reflection, in this paper we establish uniform-in-time estimates on probability distances for mean-field type SDEs, where the drift terms under consideration are dissipative merely in the long distance. As applications, we (i) explore the long time probability distance estimate between an SDE and its delay version; (ii) investigate the issue on uniform-in-time propagation of chaos for McKean-Vlasov SDEs, where the drifts might be singular with respect to the spatial variables and need not to be of convolution type; (iii) tackle the discretization error bounds in an infinite-time horizon for stochastic algorithms (e.g. backward/tamed/adaptive Euler-Maruyama schemes as three typical candidates) associated with McKean-Vlasov SDEs.

### Interlacing polynomial method for matrix approximation

JIANFENG CAI

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Matrix approximation is a crucial technique in numerous research areas across science and engineering, such as machine learning, scientific computing, and signal processing. These fields often deal with high-dimensional datasets formatted as matrices, which necessitates the use of matrix approximation as a fundamental step in data processing. In this talk, we address the problem of approximating a data matrix by selecting a subset of its columns and/or rows either from the matrix itself or from other source matrices. We

apply the method of interlacing polynomials, introduced by Marcus, Spielman, and Srivastava, to develop new deterministic algorithms and establish a theoretical limit on the minimum approximation error. Our algorithm is deterministic and operates in polynomial time. Additionally, our new bounds are asymptotically sharp in several challenging scenarios where current methods provide unnecessarily large error bounds.

#### On the long time behavior of mean field control problems

ALEKOS CECCHIN University of Padova, Italy

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We study the long time behavior of controlled McKean-Vlasov dynamics with a non-degenerate idiosyncratic noise, without assuming convexity, nor monotonicity, of the cost coefficients. Instead, we consider a drift component which is strongly decreasing, but just outside a ball. Using coupling by reflection arguments, we establish uniform in time estimates for the Lipschitz constant of the value function of the mean field control problem and of its measure derivative. Thus we show existence and uniqueness of the turnpike, i.e. the stationary measure which solves the ergodic problem, and establish exponential convergence of the optimal trajectories to such equilibrium, as the time horizon grows. We also prove uniform in time propagation of chaos for the related symmetric control problem with N cooperative agents.

Based on joint work with Giovanni Conforti, Alain Durmus, and Katharina Eichinger (École Polytechnique).

### Convergence analysis of nonlinear Kaczmarz method for systems of nonlinear equations with component-wise convex mapping

CHONG CHEN

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Motivated by a class of nonlinear imaging inverse problems, for instance, multispectral computed tomography (MSCT), we study the convergence theory

of the nonlinear Kaczmarz method (NKM) for solving the system of nonlinear equations with component-wise convex mapping, namely, the function corresponding to each equation being convex. However, such kind of nonlinear mapping may not satisfy the commonly used component-wise tangential cone condition (TCC). For this purpose, we propose a novel condition named relative gradient discrepancy condition (RGDC), and make use of it to prove the convergence and even the convergence rate of the NKM with several general index selection strategies, where these strategies include cyclic strategy and maximum residual strategy. Particularly, we investigate the application of the NKM for solving nonlinear systems in MSCT image reconstruction. We prove that the nonlinear mapping in this context fulfills the proposed RGDC rather than the component-wise TCC, and provide a global convergence of the NKM based on the previously obtained results. Numerical experiments further illustrate the numerical convergence of the NKM for MSCT image reconstruction.

### A Scattering theory on hyperbolic spaces

Lu Chen

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In this talk, we introduce a theoretical framework for time-harmonic wave scattering on hyperbolic spaces. Using the limiting absorption principle (LAP), we derive the explicit forms of the ingoing and outgoing Green functions of the Helmholtz operator of hyperbolic spaces. Then we give the accurate characterisations of the asymptotic behaviours of the Green functions and use them to establish the ingoing and outgoing radiation conditions, which are analogues to the Sommerfeld radiation conditions in the Euclidean setting. Finally, we also discuss a hyperbolic Rellich's type theorem which guarantees that the scattered field as well as its far-field pattern are uniquely defined. To our best knowledge, the theoretical framework is new to the literature and it paves the way for many subsequent developments for wave scattering on hyperbolic spaces. This talk is based on the joint work with Professor Hongyu Liu from City University of Hong Kong.

### A two time-scale evolutionary game approach to multiagent reinforcement learning and its application in algorithmic collusion

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We propose a two time-scale evolutionary game approach to solving generalsum multiagent reinforcement learning (MARL) problems in this paper. Different from the existent literature that requires to solve Nash equilibrium strategies, exactly or approximately, in each episode of learning, the new approach incorporates two innovative designs. First, we propose a simple perturbed best response based protocol to update policies. This enables us to avoid the computationally expensive step of finding the exact equilibrium at each state. Second, we update agents' policies and their Q-values in two different time scales to overcome the non-stationary obstacle in learning process. The new approach provably converges to approximate Nash equilibria of MARL problems without imposing the global optima or saddle point conditions, two restrictive assumptions that are typically needed in the literature. It can be easily extended to the setups of unobservable policies and private states. As two special examples, we investigate how to apply our approach in zero-sum and cooperative MARL problems. One application in algorithmic collusion modeling is also discussed in the numerical experiment part.

### Towards high-fidelity text-to-3D generation

YINGCONG CHEN

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Recent advancements in text-to-3D generation have unlocked new avenues for creating imaginative 3D assets, yet the quest for high-quality, detailed models remains a significant challenge. A key limitation lies in the prevalent use of Score Distillation Sampling (SDS), which leads to inconsistent and low-quality updates, causing an over-smoothing effect. In this talk, I will share insights from our recent work, LucidDreamer. Particularly, I will delve into the Interval Score Matching (ISM) approach, a promising alternative of SDS that leverages deterministic diffusing trajectories and interval-based

score matching to significantly enhance the quality of 3D model generation. I will explore the technical nuances of ISM, highlighting how it diverges from and improves upon existing methodologies. The presentation will also showcase our experimental findings, demonstrating ISM's capability in producing more realistic 3D models with reduced training costs and a more streamlined training pipeline.

### Svd-based graph Fourier transforms on directed graphs

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Graph signal processing provides an innovative framework to process data on graphs. The widely used graph Fourier transform on the undirected graph is based on the eigen-decomposition of the Laplacian. In many engineering applications, the data is time-varying and pairwise interactions among agents of a network are not always mutual and equitable, such as the interaction data on a social network. Then the graph Fourier transform on directed graph is in demand and it should be designed to reflect the spectral characteristic for different directions to decompose graph signals into different frequency components, and to efficiently represent the graph signal by different modes of variation. In this talk, I will present our recent work on the graph Fourier transforms on directed graphs which are based on the singular value decompositions of the Laplacians.

### Equivalent extensions of partial differential equations on curves or surfaces

Jay Chu

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In this talk, we propose a methodology that extends the energy function defined on surfaces to the energy function defined on the nearby tubular neighborhood that gives the same energy when inputting the constant-along-normal extension. Furthermore, the extended energy function gives the same minimizer as which the original energy function gives in the sense of restriction on the surface. This new approach connects the original energy function

to an extended energy function and provides a good framework to solve PEDs numerically on Cartesian grids.

Recently, we have used the sign distance function defined in a narrow-band near the moving interface to represent the evolution of the curve. We derive the equivalent evolution equations of the distance function in the narrowband. The novelty of the work is to determine the equivalent evolution equation on Cartesian girds without extra conditions or constraints. The proposed method extends the differential operators appropriately so that the solutions on the narrowband are the distance function of the solution to the original mean flow solution. Furthermore, the extended solution carries the correct geometric information, such as distance and curvature, on Cartesian grids. Some numerical experiments confirm that the proposed method is convergent numerically. A simple example shows that the methodology can also be extended to optimal transpose problem defined on the sphere. This is a joint work with Richard Tsai, Ming-Chih Lai, Shih-Hsuan Hsu, Chun-Chieh Lin.

### Sharper guaranteed lower bounds for eigenvalues

QUANLING DENG

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In this talk, I will introduce fully computable two-sided bounds on the eigenvalues of the second-order elliptic operator on arbitrarily coarse meshes based on the nonconforming Crouzeix-Raviart finite element method. In particular, I will start with the lower bounds initially established by Carstensen and Gedicke in 2014 and then introduce several lines of ideas to improve these bounds, particularly for eigenvalues of the coarse meshes and higher eigenvalues on fine meshes. This talk is based on a joint work with Carsten Carstensen (Humboldt-Universität zu Berlin) and Tim Stiebert (Humboldt-Universität zu Berlin).

### Mathematical framework for multi-layered medium

Youjun Deng

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In this talk, we shall present some recent work on multi-layered medium. We shall build some elementary mathematical framework for multi-layer medium in electro-static and elasto-static systems. Based on the framework, we shall show some asymptotic results for multi-layered medium, which is a special case of inhomogeneous medium. We shall also show how to design multi-layer structures of metamaterials, which may greatly increase the resonance modes, by using the algebraic framework we designed.

# Visibility, invisibility and unique recovery of inverse electromagnetic problems with conical singularities

HUAIAN DIAO Jilin University, China *Email:* diao@jlu.edu.cn

In this talk, we study the time-harmonic electromagnetic scattering in two distinct scenarios: one involving anomalous scatterers represented by pairs of electromagnetic sources, and the other by inhomogeneous mediums, both possessing compact supports. Our primary focus lies in addressing the geometrical inverse scattering problem of determining the support of the scatterer, independent of its physical composition, through a single far-field measurement. We operate under the assumption that the scatterer's support (locally) exhibits a conical singularity. We establish a local characterisation of the scatterer when invisibility/transparency occurs, showing that its characteristic parameters must vanish locally around the conical point. Using this characterisation, we establish several local and global uniqueness results for the aforementioned inverse scattering problems, showing that visibility must imply unique recovery. In the process, we also establish the local vanishing property of the electromagnetic transmission eigenfunctions around a conical point under the Hölder regularity or a regularity condition in terms of Herglotz approximation.

# Global smooth solutions to the isentropic and irrotational Chaplygin gases with large data

BINGBING DING

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In this talk, I will focus on the global existence of smooth solutions to the 2D isentropic and irrotational Euler equations for Chaplygin gases with a general class of short pulse initial data, which, in particular, resolves in this special case, the Majda's conjecture on the non-formation of shock waves of solutions from smooth initial data for multi-dimensional nonlinear symmetric systems which are totally linearly degenerate. The major difficulties are caused by the slower time decay and the largeness of the solutions to the 2D quasilinear wave equation, some new auxiliary energies and multipliers are introduced to overcome these difficulties. This is a joint work with Prof. Zhouping Xin and Prof. Huicheng Yin.

### A hypocoercivity-exploiting stabilised finite element method for Kolmogorov equation

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We propose a new stabilised finite element method for the classical Kolmogorov equation. The latter serves as a basic model problem for large classes of kinetic-type equations and, crucially, is characterised by degenerate diffusion. The stabilisation is constructed so that the resulting method admits a numerical hypocoercivity property, analogous to the corresponding property of the PDE problem. More specifically, the stabilisation is constructed so that spectral gap is possible in the resulting "stronger-than-energy" stabilisation norm, despite the degenerate nature of the diffusion in Kolmogorov, thereby the method has a provably robust behaviour as the "time" variable goes to infinity. We consider both a spatially discrete version of the stabilised finite element method and a fully discrete version, with the time discretisation realised by discontinuous Galerkin timestepping. Both stability and a priori error bounds are proven in all cases. Numerical exper-

iments verify the theoretical findings.

# Well-posedness of regular solutions for 3D full compressible Navier-Stokes equations with degenerated viscosities

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We identify one class of initial data admitting a local-in-time regular solution with far field vacuum to the Cauchy problem of the three-dimensional (3-D) full compressible Navier-Stokes equations (CNS) with degenerate viscosities. Furthermore, it is shown that within its life span of such a regular solution, u stays in an inhomogeneous Sobolev space, i.e.,  $u \in H^3(\mathbb{R}^3)$ , where S has uniformly finite lower and upper bounds in the whole space, and the laws of conservation of total mass, momentum and total energy are all satisfied.

### Information theoretic learning meets deep neural networks

Jun Fan

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Information theoretic learning, a machine learning approach that incorporates ideas from information theory, offers a family of supervised learning algorithms based on the principle of minimum error entropy (MEE). These algorithms provide an alternative to traditional least squares methods, particularly effective when dealing with heavy tailed noises or outliers. The integration of information theoretic learning with deep learning has garnered significant attention in addressing the evolving challenges of modern machine learning. In this talk, we will delve into the theoretical exploration of MEE algorithms generated by deep neural networks in the context of regression tasks. We focus on establishing fast learning rates for these algorithms when the noise satisfies weak moment conditions.

### A new approach to transonic shock solutions for steady 1-D Euler-Poisson equations in an interval

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In this talk we are going to introduce a new approach to the existence of the transonic shock solutions for steady 1-D Euler-Poisson equations in an interval under certain sufficient conditions, which could be regarded as a special case of the results established by Luo and Xin in "Comm. Math. Sci. 10, (2012): 419-462" with delicate analysis on the phase space of the Euler-Poisson equations.

# Efficient preconditioners for large-scale highly heterogeneous Darcy flow

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I will first introduce a two-grid preconditioner for large-scale Darcy flow in highly heterogeneous porous media. We start from the discretization of the first-order form for the Darcy flow and apply a velocity elimination strategy to obtain an equation with pressure as the only unknown. Then an efficient preconditioner is designed to solve this equation. The key component of the two-grid preconditioner is adoption of a non-standard coarse space that contains the media's heterogeneity information. We solve a carefully constructed spectral problem in each coarse element to form the non-standard coarse space. We also extend this method for multigrid regime. The experimental results show that our generalized multiscale space based preconditioner is robust with respect to the contrast, size and geometry of the permeability fields.

#### Robust domain decomposition strategies for high-contrast multiscale problems on irregular domain using GMs-FEM

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Our research focuses on developing domain decomposition preconditioners tailored for second-order elliptic partial differential equations. We address two key challenges: handling coefficients with high contrast and accommodating irregular domains. The robustness of our preconditioners is crucial for real-world applications, such as modeling subsurface flow in porous media. Our method constructs a partition of unity using local spectral information and implements a two-level additive Schwarz preconditioner. Numerical experiments confirm their robustness. This talk follows: Calvo, Juan G., and Juan Galvis. "Robust domain decomposition methods for high-contrast multiscale problems on irregular domains with virtual element discretizations." Journal of Computational Physics (2024): 112909.

### Machine learning for inverse scattering problems

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The objective of the inverse scattering problem is to determine the geometric or physical parameters of scatterers through the observed wavefield data. Such inverse problems have broad applications in fields such as medical imaging, geophysics, materials design, etc. With the developments in artificial intelligence and big data, machine learning has made significant breakthroughs in various disciplines and applications in recent years. This talk investigates the application of machine learning to solving inverse scattering problems from the perspectives of data and model.

# Explicit constructions of pairs of dual frames on locally compact abelian groups

SAY SONG GOH

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Gabor frames and wavelet frames are two well-known systems for sparse representations of functions on the real line. They can be unified and generalized under the setting of frames on locally compact abelian (LCA) groups. This generalization covers both stationary and nonstationary cases, and also extends to variants of Gabor frames and wavelet frames on various LCA groups. The resulting Fourier-like frames are generated by modulates of a collection of functions on the dual group, and via the Fourier transform, they form a generalized shift-invariant system on the original group. Our focus is on explicit constructions of such Fourier-like frames on LCA groups and their duals. We first develop a general and flexible approach for constructing partitions of unity on LCA groups based on certain tilings of the group. Using these partitions of unity, we derive general explicit constructions of pairs of dual frames on LCA groups. This requires a supply of sufficiently fine lattices, a condition that is automatically satisfied for all elementary LCA groups. Concrete constructions of pairs of dual frames on several LCA groups of interest are provided as illustration. This is joint work with Ole Christensen.

### The coupled complex boundary methods for inverse problems of partial differential equations

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In this talk, a coupled complex boundary method (CCBM) is proposed for an inverse source problem. With the introduction of imaginary unit, the CCBM transfers the original real problem to a complex one. The CCBM has several merits and is further improved. Also, the applications of the CCBM to bioluminescence tomography, inverse Cauchy problem etc. are delivered.

### Local well-posedness for the free boundary incompressible elastodynamics via vanishing viscosity limit

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In this talk, we consider the free boundary problem of incompressible elastodynamics, a coupling system of the Euler equations for the fluid motion with a transport equation for the deformation tensor. Under a natural force balance law on the free boundary, we establish the well-posedness theory for a short time interval. Our method uses the vanishing viscosity limit by establishing a uniform a priori estimate with respect to the viscosity. As a byproduct, the inviscid limit of the incompressible viscoelasticity (the system coupled with the Navier–Stokes equations) is also justified. Based on a crucial new observation about the inherent structure of the elastic term on the free boundary, we successfully develop a new estimate for the pressure and then manage to apply an induction method to control normal derivatives. This strategy also allows us to establish the vanishing viscosity limit in standard Sobolev spaces, rather than only in the conormal ones.

# Vanishing surface tension limit of viscous surface wave equations in an infinite layer

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Consideration in this talk is a viscous fluid of finite depth below the air, occupying a 3-D horizontal infinite domain bounded below by a fixed solid boundary and above by a free moving boundary. The fluid dynamics are governed by the gravity-driven incompressible Navier-Stokes equations with the effect of surface tension on the free surface, which results in the anisotropic decay or growth of the free surface.

By using Lagrangian approach, we prove global well-posedness of the surface wave problem with small initial perturbation near equilibrium, and establish vanishing surface tension limit result. This is a joint work with Prof. Tao Luo.

### Towards a novel iterative method for the inverse elastic scattering problem

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This talk concerns the time-harmonic inverse elastic scattering problem of reconstructing the unknown rigid obstacle embedded in an open space filled with a homogeneous and isotropic elastic medium. Utilizing the near-field data and the boundary condition, a Newton-type iteration scheme is designed to identify the boundary curve of the obstacle. Based on the Helmholtz decomposition and the Fourier-Bessel expansion, we explicitly derive the approximate scattered field and its derivative on each iterative curve. Mathematical justifications for the convergence of the proposed method will be discussed. Numerical examples will be also presented to illustrate the validity of the proposed method.

### Centered kernels and pairwise learning

XIN GUO

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Pairwise learning is a large family of learning algorithms for the problems where supervised labels are not available, but one has only the access to the differences between labels of each pair of sample points. For example, ranking, AUC maximization, metric learning, gradient learning, and so on. We studied a transform of reproducing kernels so that the reproducing kernel Hilbert space defined by the obtained kernel lies in the orthogonal complement of constant functions, and is thus a perfect hypothesis space for learning the scoring functions. We proved that the kernel space complexity is invariant after this transformation. We also obtained some relations between the integral operators before and after the kernel transformation.

### Learning theory of spectral algorithms under covariate shift

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In machine learning, it is commonly assumed that the training and test samples are drawn from the same underlying distribution. However, this assumption may not always hold true in practice. In this talk, we delve into a scenario where the distribution of the input variables (also known as covariates), differs between the training and test phases. This situation is referred to as covariate shift. To address the challenges posed by covariate shift, various techniques have been developed, such as importance weighting, domain adaptation, and reweighting methods. In this talk, we specifically focus on the weighted spectral algorithm. Under mild conditions imposed on the weights, we demonstrate that this algorithm achieves satisfactory convergence rates. This talk is based on joint work with Prof. Jun Fan and Prof. Lei Shi.

### Wavelet methods for the elliptic interface problems

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Let  $\Omega \subset \mathbb{R}^2$  be a rectangular domain and  $\Gamma$  be a curve inside  $\Omega$  such that  $\Gamma$  splits  $\Omega$  into two subregions  $\Omega_+$  and  $\Omega_-$ . The elliptic interface problem considers the following PDE:

$$\begin{cases} -\nabla \cdot (a\nabla u) = f & \text{in } \Omega \backslash \Gamma, \\ [u] := u_+ - u_- = g & \text{on } \Gamma, \\ [a\nabla u \cdot \vec{n}] := [a_+ \nabla u_+ - a_- \nabla u_-] \cdot \vec{n} = g_\Gamma & \text{on } \Gamma, \\ u = g_b & \text{on } \partial \Omega, \end{cases}$$

where  $\vec{n}$  is the unit normal vector of  $\Gamma$  towards  $\Omega_-$ . Though the solution of u on each subregion  $\Omega_+$  and  $\Omega_-$  is often smooth, the solution u itself in the domain  $\Omega$  is not continuous across the interface  $\Gamma$  due to the jump condition  $[u] = g_1 \neq 0$ . Even with  $g_1 = 0$  for a continuous solution u, u often belongs

to the Sobolev space  $H^{1+\epsilon}(\Omega)$  with at most  $\epsilon < 1/2$  due to the second jump condition on flux and discontinuity of the diffusion coefficient a across the interface  $\Gamma$ . This imposes challenges to traditional numerical methods. Currently, many modified finite difference and finite element methods have been proposed to attack elliptic interface problems. Most of these methods require complicated treatment along the interface and lack uniformly bounded condition numbers. In this talk, we will propose a (nonclassical but simple) wavelet method to solve the elliptic interface problems. First, our wavelet method avoids complicated treatments along the interface and only need the information about whether the support of a wavelet element touches the interface  $\Gamma$  or not (which is trivial). Second, the linear system has uniformly bounded condition numbers and our method is essentially a meshfree method. Third, regardless of the low regularity  $u \in H^{1+\epsilon}(\Omega)$  with  $\epsilon < 1/2$ , we prove that our wavelet method achieves arbitrarily high-order convergence rates in dimension one, the second-order convergence rate in dimension two, and the convergence rate of order 1.75 in dimension three. Numerical experiments will be provided to demonstrate the theoretical convergence rates of our proposed wavelet methods for dimensions one and two. This is joint work with M. Michelle at Purdue University.

# Surface wave scattering problems in a periodic inhomogeneous layer with local defects

Guanghui Hu Nankai University, Tianjin, China *Email:* ghhu@nankai.edu.cn

Let  $W_h := \mathbb{R} \times (0, h) \subset \mathbb{R}^2$  be an open waveguide characterized by a refractive index function which is  $2\pi$ -periodic in the horizontal direction. Assume further that  $W_h$  is sitting on a perfectly conducting plate and the medium above is homogeneous and isotropic. We consider time-harmonic scattering of surface waves (which are also called guided/Floquet waves) from a local defect embedded inside  $W_h$ . In this talk we shall discuss properties of surface waves and establish a new radiation condition for time-harmonic wave scattering problems in an open periodic waveguide in two dimensions. Uniqueness and existence of solutions of the mathematical model will be shown.

### Characteristic block-centered finite difference methods for Darcy-Forchheimer compressible miscible displacement problem

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In this talk, we present characteristic block-centered finite difference methods for solving the nonlinear Darcy-Forchheimer compressible miscible displacement problem in porous media. The block-centered finite difference method is used to discretize the miscible problem, where the pressure equation is described by the nonlinear Darcy-Forchheimer model, and the transport equation is addressed with the help of the characteristic method. Two-grid methods are developed for the nonlinear system. The nonlinear system is linearized using Newton's method with a small positive parameter to ensure the differentiability of the nonlinear term in the Darcy-Forchheimer equation. A modified two-grid algorithm is proposed to further reduce the computational cost of the time-dependent problem. The proposed methods are rigorously analyzed, and a priori error estimates are provided for the rates of convergence of the velocity, pressure, concentration, and its flux. Finally, numerical experiments are conducted to demonstrate the effectiveness of the proposed methods by comparing the efficiency with that of other solvers, especially in terms of CPU time.

# On the optimal order approximation of the partition of unity finite element method

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In the partition of unity finite element method, the nodal basis of the standard linear Lagrange finite element is multiplied by the  $P_k$  polynomial basis to form a local basis of an extended finite element space. Such a space contains the  $P_1$  Lagrange element space, but is a proper subspace of the  $P_{k+1}$ Lagrange element space on triangular or tetrahedral grids. It is believed that the approximation order of this extended finite element is k, in  $H^1$ -norm, as it was proved in the first paper on the partition of unity, by Babuska and Melenk and this space does not even contain the  $C^0$ - $P_k$  space. In this talk, we show surprisingly the approximation order is k+1 in  $H^1$ -norm. In addition we extend the method to rectangular/cuboid grids and give a proof to this sharp convergence order.

This is a joint work with Shangyou Zhang, University of Delaware.

### Neural expectation maximization for self-supervised blind image deblurring

Hui Ji

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When taking a picture, any camera shake during the shutter time can result in a blurred image. Recovering a sharp image from the one blurred by camera shake is a challenging yet important problem. Most existing deep learning methods use supervised learning to train a deep neural network (DNN) on a dataset of many pairs of blurred/latent images. In this talk, we will present a dataset-free deep learning method for removing uniform and non-uniform blur effects from images of static scenes. Based on a DNN-based re-parametrization of the latent image and blur kernels, a Monte Carlo Expectation Maximization (MCEM) approach is presented to train the DNN without requiring any latent images. The Monte Carlo simulation is implemented via Langevin dynamics. Experiments showed that the proposed method outperforms existing methods significantly in removing motion blur from images of static scenes.

### A PML method for signal-propagation problems in axon

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This talk is focused on the modelling of signal propagations in myelinated axons. The well-posedness of model is established upon Dirichlet boundary conditions at the two ends of the neural structure and the radiative condition

in the radial direction of the structure. Using the perfectly matched layer (PML) method, we truncate the unbounded background medium and propose an approximate problem on the truncated domain. The well-posedness of the PML problem and the exponential convergence of the approximate solution to the exact solution are established. Numerical experiments are presented to demonstrate the theoretical results and the efficiency of our methods to simulate the signal propagation in axons.

#### Spectral patterns of elastic transmission eigenfunctions: boundary localisation, surface resonance and stress concentration

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We present a comprehensive discovery on the spectral patterns of elastic transmission eigenfunctions, including boundary localisation, boundary resonance and stress concentration. In the case with radial geometry and constant coefficients, we give rigorous justifications and derive thorough understanding of those intriguing geometric and physical patterns. We also present numerical examples to verify that the same results hold in general geometric and parameter setups.

### On the early stopping of untrained convolutional neural networks

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In recent years, new regularization methods based on (deep) neural networks have shown very promising empirical performance for the numerical solution of ill-posed problems. Due to the nonlinearity of neural networks, these methods often lack satisfactory theoretical justification. In this talk, we rigorously discuss the convergence of one unsupervised approach utilizing untrained convolutional neural networks to represent solutions to linear ill-posed problems. The regularization property of the approach relies solely on the architecture

of the neural network instead. We shall show that the classical discrepancy principle is an adequate method for early stopping of two-layer untrained convolutional neural networks learned by gradient descent, and furthermore, it yields an approximation with minimax optimal convergence rates. Numerical results are also presented to illustrate the theoretical findings.

### Homogenisation of discrete dynamical optimal transport

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We consider the large-scale behaviour of dynamical optimal transport on  $\mathbb{Z}^d$ -periodic graphs. We obtain a homogenisation result that describes the effective behaviour of the discrete problems in terms of a continuous optimal transport problem.

### Hypersonic similarity for steady hypersonic flow over two-dimensional Lipschitz wedge

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In this talk, I will present our recent results on the mathematical validation of hypersonic similarity law for steady hypersonic flow over a two-dimensional Lipschitz wedge and the convergence rates are also present for both potential flow and full Euler flow under the framework of  $BV \cap L^1$ .

# The singularity swapping method for boundary integral equations and its applications to wave scattering

Jun Lai

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Accurate evaluation of nearly singular integrals plays an important role in

many boundary integral equation based numerical methods. In this talk, we propose a variant of singularity swapping method to accurately evaluate the layer potentials for arbitrarily close targets. Our method is based on the global trapezoidal rule and trigonometric interpolation, resulting in an explicit quadrature formula. The method achieves spectral accuracy for nearly singular integrals on closed analytic curves. Numerical examples for Laplace's and Helmholtz equations show that high order accuracy can be achieved for arbitrarily close field evaluation. Extension to three dimensional singular integrals will also be discussed.

### A feature-capturing PINN for Stokes problems with discontinuous viscosity and singular forces

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In this talk, we present a discontinuity and cusp capturing physics-informed neural network (PINN) to solve Stokes equations with a piecewise constant viscosity and singular force along an interface. We first reformulate the governing equations in each fluid domain separately and replace the singular force effect by the traction balance equation between solutions in two sides of the interface. Since the pressure is discontinuous and the velocity has discontinuous derivatives across the interface, we hereby use a network consisting of two fully-connected sub-networks that are able to capture the pressure and velocity behaviors across the interface sharply. The numerical results in 2D and 3D show an excellent accuracy performance comparing with traditional immersed interface methods in literature.

### Generalization and optimization of gradient methods for shallow neural networks

Yunwen Lei

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Neural networks have achieved impressive performance in various applications. In this talk, we discuss the generalization and optimization of shallow neural networks (SNNs). We consider both gradient descent (GD) and stochastic gradient descent (SGD) to train SNNs. We show how the generalization and optimization should be balanced to obtain consistent error bounds under a relaxed overparameterization setting. We improve the existing estimates on the weak-convexity parameter of SNNs along the trajectories of optimization process.

#### Energy cascade of Klein-Gordon

ZHEN LEI

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We will present our recent studies on the energy cascade theory of nonlinear Klein-Gordon equations. Energy moves from discrete modes to continuous modes. The mechanism and the lower and upper bounds of energy transfer will be presented.

#### Multicontinuum homogenization and its application

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In this talk, we will explore the recent advancements in multiscale simulations for high contrast problems with no scale separation. We will specifically focus on the nonlocal multicontinua (NLMC) method, which introduces multiple macroscopic variables in each computational grid. We present a general derivation of multicontinuum equations and discuss cell problems. We consider the problem with distinct properties in different regions, which gives distinct macroscopic equations for those regions. We will connect the macroscopic equations by interface conditions. We present constraint cell problem formulations in a representative volume element with an oversampling technique to obtain the multicontinuum equations and interface conditions.

### Some work on the analysis of equations of fluid type

Congming Li

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In this talk, we revisit some ideas behind our earlier work on the analysis of incompressible fluids. Then we present some recent work on localized a priori estimate of solutions to incompressible Navier-Stokes equations. Weak-strong uniqueness type analysis will also be discussed.

# Wavelet-based edge multiscale finite element methods for singularly perturbed convection-diffusion equations

Guanglian Li

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We propose a novel efficient and robust Wavelet-based Edge Multiscale Finite Element Method (WEMsFEM) to solve the singularly perturbed convectiondiffusion equations. The main idea is to first establish a local splitting of the solution over a local region by a local bubble part and local Harmonic extension part, and then derive a global splitting by means of Partition of Unity. This facilitates a representation of the solution as a summation of a global bubble part and a global Harmonic extension part, where the first part can be computed locally in parallel. To approximate the second part, we construct an edge multiscale ansatz space locally with hierarchical bases as the local boundary data that has a guaranteed approximation rate without higher regularity requirement on the solution. The key innovation of this proposed WEMsFEM lies in a provable convergence rate with little restriction on the mesh size or the regularity of the solution. Its convergence rate with respect to the computational degree of freedom is rigorously analyzed, which is verified by extensive 2-d and 3-d numerical tests. This is a joint work with Eric Chung (CUHK, Hong Kong) and Shubin Fu (Eastern Institute of Technology, P.R. China).

### Babuška problem in composite materials: PDE analysis and FEM

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In high-contrast composite material, the small distance between inclusions always leads to the stress concentration and then initiate damage. For Babuška problem in the context of linear elasticity, we established blow-up asymptotic expressions in the narrow region between adjacent inclusions for the gradients and second order derivative estimates of the solutions, as the distance tends to zero. Based on these estimates, a kind of new FEM are developed and several numerical experiments are designed.

#### The effective construction on elastic metamaterials

Hongjie Li

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In this talk, different kinds of structures are explored to effectively achieve the negative elastic metamaterials. First, the structure involving the bubbles embedded within the soft elastic materials will be discussed, offering an effective means to achieve the negative bulk modulus. Then the configuration incorporating hard inclusions embedded in the soft elastic matrix shall be investigated, enabling the attainment of negative values in the mass density.

# A priori error estimate of deep mixed residual method for elliptic PDEs

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In this work, we derive a priori error estimate of the deep mixed residual method (DMRM) when solving some elliptic partial differential equations (PDEs) using neural networks. DMRM is a new deep-learning based method

for solving PDEs and it has been shown to be efficient and accurate in previous studies. Our work is the first theoretical study of DMRM. We prove that the neural network solutions will converge if we increase the training samples and network size without any constraint on the ratio of training samples to the network size. Besides, our results suggest that the DMRM can approximate the Laplacian of the solution by the intermediate auxiliary variable, which is dismissing in the deep Ritz method (DRM). It is also verified by the numerical experiments.

### A novel boundary integral formulation for the biharmonic wave scattering problem

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The talk is concerned with the cavity scattering problem associated with biharmonic waves in an infinite thin plate. Based on the operator splitting, the scattering problem is recast into a coupled boundary value problem for the Helmholtz and modified Helmholtz equations. A novel boundary integral formulation is proposed for the coupled problem. The convergence is analyzed for the semi- and full-discrete schemes of the boundary integral system. Numerical results show that the proposed method is highly accurate for both smooth and nonsmooth examples.

### Smoothing algorithms for nonsmooth optimization over the Stiefel manifold with applications to the graph Fourier basis problem

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In this talk, we consider a class of nonsmooth and nonconvex optimization problems over the Stiefel manifold where the objective function is the summation of a nonconvex smooth function and a nonsmooth Lipschitz continuous convex function composed with a linear mapping. Besides, we are interested in its application to the graph Fourier basis problem. We propose three numerical algorithms for solving this problem, by combining smoothing methods and some existing algorithms for smooth optimization over the Stiefel manifold. In particular, we approximate the aforementioned nonsmooth convex function by its Moreau envelope in our smoothing methods, and prove that the Moreau envelope has many favorable properties. Thanks to this and the scheme for updating the smoothing parameter, we show that any accumulation point of the solution sequence generated by the proposed algorithms is a stationary point of the original optimization problem. Numerical experiments on building graph Fourier basis are conducted to demonstrate the efficiency of the proposed algorithms.

### Model reduction methods based on DMD for parametric dynamical systems

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Parametric dynamical systems are widely used to model physical systems, but their numerical simulation can be computationally demanding due to nonlinearity, long-time simulation, and multi-query requirements. Model reduction methods aim to reduce computation complexity and improve simulation efficiency. However, traditional model reduction methods are inefficient for parametric dynamical systems with nonlinear structures. To address this challenge, we propose an adaptive method based on local dynamic mode decomposition (DMD) to construct an efficient and reliable surrogate model. We propose an improved greedy algorithm to generate the atoms set  $\Theta$  based on a sequence of relatively small training sets, which could reduce the effect of large training sets. At each enrichment step, we construct a local subsurrogate model using the Taylor expansion and DMD, resulting in the ability to predict the state at any time without solving the original dynamical system. Moreover, our method provides the best approximation almost everywhere over the parameter domain with certain smoothness assumptions, thanks to the gradient information. At last, three concrete examples are presented to illustrate the effectiveness of the proposed method.

### Exploring structural sparsity of coil images from 3-dimensional directional tight framelets for SENSE reconstruction

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Each coil image in a parallel magnetic resonance imaging (pMRI) system is an imaging slice modulated by the corresponding coil sensitivity. These coil images, structurally similar to each other, are stacked together as a 3dimensional (3D) image data and their sparsity property can be explored via 3D directional Haar tight framelets. The features of the 3D image data from the 3D framelet systems are utilized to regularize sensitivity encoding (SENSE) pMRI reconstruction. Accordingly, a so-called SENSE3d-algorithm is proposed to reconstruct images of high quality from the sampled K-space data with a high acceleration rate by decoupling effects of the desired image (slice) and sensitivity maps. Since both the imaging slice and sensitivity maps are unknown, this algorithm repeatedly performs a slice-step followed by a sensitivity-step by using updated estimations of the desired image and the sensitivity maps. In the slice-step, for the given sensitivity maps, the estimation of the desired image is viewed as the solution to a convex optimization problem regularized by the sparsity of its 3D framelet coefficients of coil images. This optimization problem, involved data from the complex field, is solved by a primal-dual-three-operator splitting (PD3O) method. In the sensitivity-step, the estimation of sensitivity maps is modelled as the solution to a Tikhonov-type optimization problem that favours the smoothness of the sensitivity maps. This corresponding problem is nonconvex, and could be solved by a forward-backward splitting method. Experiments on real phantoms and in-vivo data show that the proposed SENSE3d-algorithm can explore the sparsity property of the imaging slices and efficiently produce reconstructed images of high quality with reducing aliasing artifacts caused by high acceleration rate, additive noise, as well as the inaccurate estimation of each coil sensitivity. To provide a comprehensive picture of the overall performance of our SENSE3d model, we provide quantitative index (HaarPSI) and comparisons to some deep learning methods such as VarNet and fastMRI-UNet.

### Integral operator approaches for spherical data fitting

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For kernel interpolation of scattered data on spheres, it is well known that the attainable approximation error and the condition number of the underlying interpolation matrix cannot be small simultaneously, which is referred to be "uncertainty phenomenon" for kernel interpolation, an undesirable consequence of which is that kernel interpolation is susceptible to noisy data. In this talk, we aim at developing a novel integral operator approach for deterministic sampling and proposing several remedies for the "uncertainty phenomenon". Based on the integral operator approach, we prove that the popular spectral regularization, distributed learning and random sketching are feasible to circumvent the "uncertainty phenomenon". We also present numerical simulation results showing that our mitigation methods are practical and robust in terms of handling noisy data from challenging computing environments.

# Asymptotic stability for n-dimensional magnetohydro-dynamic equations

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This talk is concerned with the stability theory of n-dimensional incompressible and compressible magnetohydrodynamic (MHD for short) equations with only kinematic viscosity or magnetic diffusion in the periodic domain  $\mathbb{T}^n$ . I will present some new results on the asymptotic stability and sharp decay estimates of this system when the magnetic field close to an equilibrium state satisfying the Diophantine condition. In the present works, by exploiting and effectively utilizing the structure of perturbation system, a new dissipative mechanism is found out and applied so that we can sharply improve the spaces of existing works, where the decay estimates and asymptotic stability of solutions are taking place. Some key ideas of our method will be discussed. This talk is based on joint works with Quansen Jiu and Yaowei Xie.

# The inverse problems for a DCIS model with free boundaries in mathematical biology

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In this talk, we are concerned with the mathematical study of a DCIS model which arises in characterizing the biological development of breast cancer. A salient feature of a DCIS model is the presence of free boundaries for describing the tumor growth which is not known in advance. We will present an iterative finite difference method for the forward problem and show that the method is of 2nd order in both space and time. For the inverse problem of recovering the nutrient consumption rate by the incisional biopsy data, we will present the unique identifiability of the inverse problem and state a novel reconstruction scheme based on a certain integral formulation. Extensive numerical experiments are conducted to corroborate the theoretical findings.

### Inverse scattering with multifrequency far field patterns at sparse observation directions

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Inverse scattering merits numerous applications in medical imaging, geophysics, non-destructive testing, and many others. Most of the work in the literature consider the case when the measurements are available all around the unknown object. However, in applications where measurements are only available at a limited number of sparse receivers, we are led to the inverse problem with sparse measurements. In this talk, we share you with our recent studies in this direction.

#### A high-accuracy mode solver for acoustic scattering by a periodic array of axially symmetric obstacles

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This talk is concerned with guided modes of an acoustic wave propagation problem on a periodic array of axially symmetric obstacles. A guided mode refers to a quasi-periodic eigenfield that propagates along the obstacles but decays exponentially away from them in the absence of incidences. Thus, the problem can be studied in an unbound unit cell due to the quasi-periodicity. We truncate the unit cell onto a cylinder enclosing the interior obstacle in terms of utilizing Rayleigh's expansion to design an exact condition on the lateral boundary. We derive a new boundary integral equation (BIE) only involving the free-space Green function on the boundary of each homogeneous region within the cylinder. Due to the axial symmetry of the boundaries, each BIE is decoupled via the Fourier transform to curve BIEs and they are discretized with high-accuracy quadratures. With the lateral boundary condition and the side quasi-periodic condition, the discretized BIEs lead to a homogeneous linear system governing the propagation constant of a guided mode at a given frequency. The propagation constant is determined by enforcing that the coefficient matrix is singular. The accuracy of the proposed method is demonstrated by a number of examples even when the obstacles have sharp edges or corners.

# On a free boundary problem of 3-D compressible Euler equations coupled with a nonlinear Poisson equation

Tao Luo

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In this talk, I will discuss some results for a free boundary problem of compressible Euler Equations coupled with a nonlinear Poisson equation in 3D. The emphasis will be on identifying stability conditions under which we obtain a priori estimates on the Sobolev norms of the fluid and electric variables and bounds for geometric quantities of free surfaces. This talk is based on joint work with Trivisa and Zeng.

# Residual-based a posteriori error estimates for an hp-discontinuous Galerkin method of the biharmonic problem

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We discuss an hp-dG residual error estimator for the biharmonic problem in 2D and 3D. Upper and lower bounds are explicit in the mesh-size and polynomial degree. The lower bound is algebraically suboptimal in terms of the polynomial degree.

[1] is the first reference where an hp-dG error estimator is analyzed for fourth order problems. The reason for this is that, in the DG context, one typically needs the existence of a C1-conforming piecewise polynomial space: this does not exist, e.g., on tetrahedral meshes for sufficiently large polynomial degree. We rather use an elliptic reconstruction of the discrete solution to the  $H^2$  space and a generalised Helmholtz decomposition of the error.

The theoretical results are confirmed by numerical experiments.

#### References

[1] Z. Dong, L. Mascotto, O. J. Sutton. Residual-based a posteriori error estimates for hp-discontinuous Galerkin discretisations of the biharmonic problem. SIAM *Journal on Numerical Analysis*,59(3):1273-1298, 2021.

#### Geometric functionals of smooth Gaussian fields

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Smooth Gaussian fields are widely used for modelling phenomena across scientific disciplines (e.g. in cosmology, medical imaging, quantum chaos and machine learning). The geometric properties of such fields are valuable for statistical testing and predictions. In this talk I will discuss recent progress

in proving central limit theorems for a number of geometric functionals associated with Gaussian fields.

#### On free boundary problems of compressible viscoelastic fluid with or without surface tension

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In this talk, we consider the free boundary problem of compressible isentropic neo-Hookean viscoelastic fluid equations with or without surface tension. Under the physical kinetic and dynamic conditions proposed on the free boundary with surface tension and some additional structure condition on the free boundary without surface tension, we investigate the regularity of classical solutions to viscoelastic fluid equations in Sobolev spaces which are uniform in viscosity and justify the corresponding vanishing viscosity limits. These are joint works with Prof. Xumin Gu from Shanghai University of Finance and Economics.

# Mathematical analysis of therapy modalities using acoustic cavitation

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We analyse the ultrasound waves reflected by multiple bubbles in the linearized time-dependent acoustic model. This study is inspired by a treatment method that employs sound wave cavitation, created by injecting bubbles into a specific area. The aim is to apply just the right amount of pressure to eliminate abnormalities in that area. We show that, at that small distance, the dominating field is reminiscent to the wave created by a point-like obstacle modeled formally by a Dirac-like heterogeneity with support at the location of the bubble and the contrast between the bubble and background material as the scattering coefficient. The mathematical analysis is done using time-domain integral equations and asymptotic analysis techniques. A well known feature here is that the contrasting scales between the bubble

and the background generate resonances (mainly the Minnaert one) in the time-harmonic regime. Such critical scales, and the generated resonances, are also reflected in the time-domain estimation of the acoustic wave. In particular, reaching the desired amount of pressure near the location of the bubble is possible only with such resonating bubbles.

#### References:

- 1. A. Mukherjee AND M. Sini, Acoustic Cavitation using Resonating Micro-Bubbles. Analysis in the Time-Domain, SIAM Journal on Mathematical Analysis, 55 (2023), pp. 5575–5616, https://doi.org/10.1137/22M1533396.
- 2. A. Mukherjee AND M. Sini, Time-Dependent Acoustic Waves Generated by Multiple Resonant Bubbles: Application to Acoustic Cavitation, Submitted, (2023), https://arxiv.org/abs/2307.12319.

#### Controllability and stochastic PDEs

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In this talk, I will review some recent results establishing a link between the long-time behavior of stochastic PDEs (ergodicity, large deviations, etc.) and the controllability of their deterministic counterparts.

# Density interpolation methods for boundary/volume integral equations in wave phenomena

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Boundary and volume integral equation methods are powerful techniques for solving linear partial differential equations with known fundamental solutions. They effectively address unbounded domains and radiation conditions at infinity, and lead to reduced-size linear systems, which despite their dense nature, can be efficiently solved using Krylov-subspace solvers coupled with Fast Multipole or  $\mathcal{H}$ -matrix algorithms. Furthermore, in wave propagation

problems, they are "free" from numerical dispersion errors. However, as is well known, the practical implementation of these methods involves evaluating challenging singular integrals.

In this talk, I will give an overview of a class of general regularization techniques aimed at numerically evaluating weakly singular, singular, hypersingular, and nearly singular boundary and volume integral operators associated with the Helmholtz and time-harmonic Maxwell's equations. These techniques address longstanding issues related to efficiency, accuracy, and practical implementation that have limited the applicability of integral equation methods in applied fields. By relying on Green's third identity and local interpolations of density functions, these techniques regularize the singularities present in integral operators, recasting them in terms of integrands that are bounded or even more regular depending on the density interpolation order. The resulting integrals can then be accurately and efficiently evaluated using elementary off-the-shelf quadrature rules. A variety of numerical examples will be presented to demonstrate the effectiveness of these techniques in the context of Nyström methods for both boundary integral equations as well as Lippmann-Schwinger-type volume integral equations.

#### Probabilistic representation of the solutions of nonlocal partial differential equations

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We propose a tree-based probabilistic representation for the solutions of semilinear partial differential equations with pseudo-differential generators, that contain the fractional Laplacian in dimension  $d \geq 1$  as a particular case. This probabilistic representation applies to the solution of the equation itself, as well as to its partial derivatives by associating one of d marks to the initial tree branch. Gradient terms are dealt with by integration by parts and subordination of Brownian motion using a Lévy process. We also apply this approach to the existence of viscosity solutions of fractional semilinear elliptic PDEs of index  $s \in (1/2, 1)$  with polynomial gradient nonlinearities on d-dimensional balls. Monte Carlo numerical experiments are performed in dimension up to d = 10.

#### Regularized mean field optimization with application to neural networks

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Our recent research focuses on advancing the field of regularized mean field optimization, with the overarching goal of establishing a theoretical foundation for evaluating the effectiveness of the training of neural networks and inspiring novel training algorithms. In this presentation, we will provide a comprehensive overview of the McKean-Vlasov dynamics, which serves as gradient flows, approaching the minimizer of regularized mean field optimization. We will place particular emphasis on examining the long-time behavior and the particle approximation of such McKean-Vlasov dynamics. Besides the gradient flows, we also introduce and investigate alternative algorithms, such as our recent work on the self-interaction diffusion, to search for the optimal weights of neural networks. Each of these algorithms is ensured to have exponential convergence, and we will showcase their performances through simple numerical tests.

#### Equations on Wasserstein space and applications

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The purpose of this talk is to give an overview of recent work involving differential equations posed on spaces of probability measures and their use in analyzing controlled multi-agent systems. The study of such systems has seen increased interest in recent years, due to their ubiquity in applications coming from macroeconomics, social behavior, and telecommunications. When the number of agents becomes large, the model can be formally replaced by one involving a mean-field description of the population, analogously to similar models in statistical physics. Justifying this continuum limit is often nontrivial and is sensitive to the type of stochastic noise influencing the population, i.e. idiosyncratic or systemic. We will describe settings for which the convergence to mean field stochastic control problems can be resolved through the

analysis of a certain Hamilton-Jacobi-Bellman equation posed on Wasserstein spaces. In particular, we develop new stability and regularity results for the equations. These allow for new convergence results for more general problems, for example, zero-sum stochastic differential games of mean-field type. We conclude with a discussion of some further problems for which the techniques for equations on Wasserstein space may be amenable.

# Stability of the Frank Wolfe algorithm for compressible signals

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The Frank Wolfe algorithm and its variations are very practical for large scale optimization problems appearing in signal processing, machine learning, and image restorations. We are concerned with the performance of the Frank Wolfe algorithm in the framework of sparse representations. We provide a unified stability analysis of the Frank Wolfe algorithm applied to constrained least squares. Moreover, we show a sharp restricted isometry property condition for the support recovery of sparse vectors in relatively noise free environments. Several numerical examples are shown to verify the accuracy and stability of the Frank-Wolfe algorithm for sparse recovery problems. The idea in this talk can also be applied to other constrained problems such as phase retrieval.

#### Classification with deep neural networks

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Deep neural networks (DNNs) trained with the logistic loss (also known as the cross entropy loss) have made impressive advancements in various binary classification tasks. Despite the considerable success in practice, generalization analysis for binary classification with deep neural networks and the logistic loss remains scarce. The unboundedness of the target function for the logistic loss in binary classification is the main obstacle to deriving satisfying generalization bounds. In this talk, we aim to fill this gap by developing a novel theoretical analysis and using it to establish tight generalization bounds for training fully connected ReLU DNNs with logistic loss in binary classification. Our generalization analysis is based on an elegant oracle-type inequality which enables us to deal with the boundedness restriction of the target function. Using this oracle-type inequality, we establish generalization bounds for fully connected ReLU DNN classifiers trained by empirical logistic risk minimization with respect to i.i.d. samples of size n, which lead to sharp rates of convergence as  $n \to \infty$ . In particular, we obtain optimal convergence rates (up to some logarithmic factor) only requiring the Hölder smoothness of the conditional class probability  $\eta$  of data. Moreover, we consider a compositional assumption that requires  $\eta$  to be the composition of several vector-valued multivariate functions of which each component function is either a maximum value function or a Hölder smooth function only depending on a small number of its input variables. Under this assumption, we can even derive optimal convergence rates for DNN classifier (up to some logarithmic factor) which are independent of the input dimension of data. This result explains why in practice DNN classifiers can overcome the curse of dimensionality and perform well in high-dimensional classification problems. Besides the novel oracle-type inequality, the sharp convergence rates presented in our talk also owe to a tight error bound for approximating the natural logarithm function near zero (where it is unbounded) by ReLU DNNs. In addition, we justify our claims for the optimality of rates by proving corresponding minimax lower bounds. All these results are new in the literature and will deepen our theoretical understanding of classification with deep neural networks.

# Convection-diffusion equation: An axiomatized framework for neural networks

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Bridging neural networks with partial differential equations holds significant importance, as it not only enhances the interpretability of neural networks but also sheds light on designing network architectures. This paper establishes convection-diffusion equation models based on rigorous theoretical analysis. The convection-diffusion equation model not only covers existing

network structures, but also illuminates novel network design, **CO**nvection dIffusion Networks (COIN). Numerical results demonstrate the effectiveness of COIN in various benchmarks, as well as its potential in novel tasks such as disease prediction.

#### Dyson Brownian motion as a Wasserstein gradient flow

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The Dyson Brownian motion (DMB) is a system of infinitely many interacting Brownian motions with logarithmic interaction potential, which was introduced by Freeman Dyson '62 in relation to the random matrix theory. In this talk, we reveal that an infinite-dimensional differential structure induced by the DBM has a Bakry-Émery lower Ricci curvature bound. As an application, we show that the DBM can be realised as the unique Wasserstein-type gradient flow of the Boltzmann-Shannon entropy associated with sine\_beta ensemble.

#### Mean field games in discrete time

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In this talk we discuss mean field games in discrete time on general probability spaces. Using dynamic programming and a forward-backward algorithm, we will construct mean field equilibria of multi period models as concatenation of equilibria of one-step games. We will also present results on convergence of discrete time games to continuous time counterparts. The talk is based on a joint work with J. Dianetti, M. Nendel and S. Wang.

#### Multiscale model order reduction for network models

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Multiphysics multiscale models on large networks are important as they appear in many applications, for example, pore network models in reservoir simulation, network models of epidemics, medical applications such as multiscale multidimensional simulations of blood flow, fibrous materials, traffic flow simulations, electric power systems, river flow models, and many others. Despite eliminating significant complexity through a network approach, solving the resulting model remains a common challenge.

We present the construction of the multiscale model order reduction for spatial networks based on the Generalized Multiscale Finite Element Method (GMsFEM). By solving local spectral problems in sub-networks, an accurate coarse-scale approximation is constructed. Numerical results are presented for square and random heterogeneous pore-network models.

#### Inverse problems for diffusion equations

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In this talk, we will show our recent results on inverse problems for diffusion equations. First, we consider an inverse boundary value problem and propose two domain sampling type methods. Second, we consider an inverse problem of simultaneously recovering the initial value and source strength. A conditional stability is proved, and a numerical algorithm is proposed. Finally, we discuss an inverse problem of simultaneously recovering the initial value and diffusion coefficient. We derive a conditional stability and propose a novel algorithm.

#### Uniform regularity and vanishing dissipation limit for the 3D magnetic Bénard equations in half space

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In this paper, we are concerned with the uniform regularity and zero dissi-

pation limit of solutions to the initial boundary value problem of 3D incompressible magnetic Bénard equations in the half space, where the velocity field satisfies the no-slip boundary conditions, the magnetic field satisfies the perfect conducting boundary conditions, and the temperature satisfies either the zero Neumann or zero Dirichlet boundary condition. With the assumption that the magnetic field is transverse to the boundary, we establish the uniform regularity energy estimates of solutions as both viscosity and magnetic diffusion coefficients go to zero, which means there is no strong boundary layer under the no-slip boundary condition even the energy equation is included. Then the zero dissipation limit of solutions for this problem can be regarded as a direct consequence of these uniform regularity estimates by some compactness arguments.

#### Radial PML-type techniques for wave scattering problems: Real versus complex coordinate transformations

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The perfectly matched layer (PML) is a well-known artificial absorbing layer commonly used to truncate computational regions of wave problems in an open unbounded domain to a finite region, and then simulate the reduced problems numerically. Although much has been known about PML, we shall propose two new ideas using both real and complex radial coordinate transformations and illustrate the essence in the time-harmonic setting. Particularly, we shall report a very recent work using real coordinate transformation that leads to a much simpler form of the PML equation. We shall also elaborate on the truly exact perfect absorbing layer (PAL) that we proposed in 2021. It is noteworthy that both techniques are based on transformations along the radial direction, in contrast to usual cartesian coordinate based PML.

# The interplay between deep learning and model reduction

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The integration of Reduced Order Modeling (ROM) and Deep Learning (DL) presents a promising avenue for enhancing computational efficiency and predictive accuracy. ROM techniques reduce complex systems into low-dimensional representations, preserving essential dynamics, while DL methodologies excel at learning intricate patterns from raw data. By combining ROM with DL, we aim to develop approaches that inherit merits from both disciplines. On one hand, we explore leveraging ROM as a preprocessing step to train DNNs with limited labeled data, addressing data scarcity. On the other hand, we intend to utilize DL to expedite ROM construction and learn ROMs directly from observational data. In this talk, we will elaborate on specific efforts made along this line.

#### Quasi-neutral limit and the boundary layer problem of Planck-Nernst-Poisson-Navier-Stokes equations for electrohydrodynamics

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In this talk, we discuss the quasi-neutral limit problem of Planck-Nernst-Poisson-Navier-Stokes equations for electro-hydrodynamics and review the known main progress on rigorously establishing quasi-neutrality theory from the view of mathematics. Quasi-neutrality is one basic assumption in physics, such as semiconductors and plasma, which was first proposed by W.Van Roosbroech in Bell System Tech. J., 1950. Firstly, we introduce some multi-scaling structure stability theory on the quasi-neutral limit and boundary layer, initial layer and mixed layer problem for drift-diffusion models in semiconductors. Then, we give some recent progress on well-posedness theory and quasi-neutral limit problem on Planck-Nernst-Poisson-Navier-Stokes equations for electro-hydrodynamics.

#### A stable and high-order hybrid numerical method for wave propagation problems

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In this presentation, I will discuss a hybrid approach for simulating wave propagation governed by second-order hyperbolic partial differential equations. Our methodology entails the utilization of a high-order finite difference scheme across the majority of the computational domain, complemented by a discontinuous Galerkin method on unstructured meshes near complex geometry. We construct projection operators at the interface of these two methods to ensure the stability and optimal convergence rate of the overall discretization. Furthermore, I will discuss an unconventional technique to represent the discontinuous Galerkin method through finite difference stencils, facilitating the derivation of an apriori error estimate for the hybrid approach. This comprehensive framework offers a promising avenue for efficient and accurate simulation of wave propagation in real-world scenarios.

# Vacuum free boundary problems in ideal compressible magnetohydrodynamics

TAO WANG

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In this talk, I will present our recent results on the local well-posedness of vacuum free boundary problems in ideal compressible magnetohydrodynamics with or without surface tension.

# Asymptotic behavior of solution for the compressible Navier-Stokes equations

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In this talk, we first show the nonlinear stability result of the planar viscous shock up to a time-dependent shift for the three-dimensional compressible Navier-Stokes equations under the generic perturbations, in particular, without zero mass conditions. Next, we introduce a vanishing dissipation limiting result of one-dimensional non-isentropic Navier-Stokes equations with shock data.

#### On the asymptotic behavior of steady Euler equations

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In this talk, we will start with the results on low Mach number limit which reveal how the external forces affect the convergence rates of the flows at far fields. And, the far field convergence rates of both incompressible and compressible flows at far fields as the boundary of the nozzle goes to flat even when the forces do not admit convergence rates at far fields. Then, for the unbounded domain, the maximum principle is applied to estimate the potential function, by choosing the proper compared functions. The convergence rates of velocity at the far field are obtained by the weighted Schauder estimates. Furthermore, we construct the examples to show the optimality of our convergence rates, and show the expansion of the incompressible airfoil flow at infinity, which indicates the higher convergence rates. We also will mention the recent work on the far-field convergence rate and stability of steady Euler flows with large vorticity and characteristic discontinuities in arbitrary infinitely long nozzles. This talk is based on the joint works with Mingjie Li, Lei Ma, Wei Xiang, Chunjin Xie, Jiaojiao Zhang.

# Global well-posedness of compressible viscous surface waves

YANJIN WANG Xiamen University, China Email: yanjin\_wang@xmu.edu.cn

We will present our recent result on the global well-posedness and stability of the free boundary problem for the compressible Navier–Stokes equations near equilibrium. The emphasis will be on the case with gravity and without surface tension when the fluid domain is horizontally infinite.

#### Partially explicit splitting scheme with explicit-implicitnull method for nonlinear multiscale flow problems

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In this talk, we present an efficient approach to solve nonlinear high-contrast multiscale flow problem. We incorporate the explicit-implicit-null (EIN) method to separate the original nonlinear term into a linear term and a damping term, and then utilise the implicit and explicit time marching scheme for the two parts respectively. Due to the multiscale property of the linear part, we further introduce a temporal partially explicit splitting scheme and construct suitable multiscale subspaces to speed up the computation. The approximated solution is splitted into these subspaces associated with different physics. The temporal splitting scheme employs implicit discretization in the subspace with small dimension that representing the high-contrast property and uses explicit discretization for the other subspace. We exploit the stability and convergence of the proposed scheme and give the condition for the choice of the linear diffusion coefficient. Numerical tests are performed to show the efficiency and accuracy of the proposed approach.

#### Solving Inverse Problem by Diffusion Model

ZHONGJIAN WANG

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Recently, generative models have achieved remarkable success in imaging, probabilistic forecasting, and solving inverse problems. The key of a generative model lies in the specification of dynamics that flow or diffuse from some base to the target distribution. These dynamics can be learned from data by optimizing certain loss functions. In this talk, our focus will be on various designs of the dynamics to enhance their efficiency, accuracy, and stability, especially in forecasting and inverse problems encountered in non-linear PDEs and imaging. In particular, we will discuss the selection of the base, the variance of the loss function, and the incorporation of conditional information.

# Image segmentation using Bayesian inference for convex variant Mumford-Shah variational model

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The Mumford-Shah model is a classical segmentation model, but its objective function is non-convex. The smoothing and thresholding (SaT) approach is a convex variant of the Mumford-Shah model, which seeks a smoothed approximation solution to the Mumford-Shah model. The energy function in the model consists of three weighted terms and the weights are called the regularization parameters. Selecting appropriate regularization parameters is crucial to achieving effective segmentation results. In this talk, we apply a Bayesian inference approach to infer the regularization parameters and estimate the smoothed image. Experimental results show that the proposed approach is capable of generating high-quality segmentation results. Although the proposed approach contains an inference step to estimate the regularization parameters, it requires less CPU running time to obtain the smoothed image than previous methods.

# On path and distribution dependent stochastic differential equations with subdifferential equations

JING WU

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In this talk, I am going to talk about the well-posedness and limit theorems for path and distribution dependent stochastic differential equations with subdifferential operators. Forward-backward system will also be discussed.

# Stochastic maximum principle for weighted mean-field system

JIE XIONG

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We study the optimal control problem for a weighted mean-field system. A new feature of the control problem is that the coefficients depend on the state process as well as its weighted measure and the control variable. By applying variational technique, we establish a stochastic maximum principle. Also, we establish a sufficient condition of optimality. As an application, we investigate the optimal premium policy of an insurance firm for asset—liability management problem. This talk is based on a joint work with Tang.

#### Neural network SDE simulator

Toshihiro Yamada

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We show a new random number sampling algorithm for stochastic differential equations which we call the "Neural network SDE (NNSDE) simulator". NNSDE simulator can learn random variables of solutions to stochastic differential equations without any information on probability distributions. Once a model has been trained, we can simulate stochastic differential equations in a similar manner to the usual random number sampling algorithm. Numerical examples for pricing problems in financial mathematics confirm the

validity of NNSDE simulator. This talk is based on a joint work with Riu Naito.

#### Well-posedness of the free surface problem for non-Newtonian fluids between cylinders rotating at different speeds: Weissenberg effect

JIAQI YANG

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In this talk, we try to present a rigorous mathematical proof of Weissenberg effect. When a liquid fills the semi-infinite space between two concentric cylinders which rotate at different steady speeds, the different fluids will lead to a different shape. For the Newtonian fluid, the meniscus descends due to the centrifugal forces. However, for certain non-Newtonian fluid, the meniscus climbs the internal cylinder, that is so called Weissenberg effect. In the previous paper (Proc. Roy. Soc. Edinburgh Sect. A 152 (5) (2022) 1251–1276), we have studied the case of Newtonian fluid, and established a rigorous mathematical analysis theory. In this paper, we focus on the non-Newtonian fluid. By splitting the problem into a system with Stokes problem and a transport equation, we shall prove the convergence of the formal perturbation series obtained by Joseph and Fosdick in Arch. Rational Mech. Anal. 49 (1972/73) 321–380, which give a rigorous proof of the Weissenberg effect.

# Time-domain mathematical modeling, finite element simulation, and design in complex anisotropic electromagnetic metamaterials

Wei Yang

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In this talk, we present a time-domain mathematical model, finite element methods, and related numerical theories for the electromagnetic propagation problem in three-dimensional anisotropic metamaterials. The model is capable of simultaneously characterizing the linear and nonlinear properties of materials. In terms of linear properties, we employ a designed finite element method to create five types of hyperbolic metamaterials within the 50-400nm

wavelength range, and numerically verify their optical performance across a wide frequency spectrum. Additionally, by combining linear hyperbolic metamaterials with nonlinear materials, we simulate and design a nonlinear hyperbolic metamaterial, effectively enhancing the third harmonic generation of electromagnetic materials using hyperbolic materials.

# Rank-based models with common noise and pathwise entropy solutions of SPDEs

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We study the mean field limit of a rank-based model with common noise, which arises as an extension to models for the market capitalization of firms in stochastic portfolio theory. We show that, under certain conditions on the drift and diffusion coefficients, the empirical cumulative distribution function converges to the solution of a stochastic PDE. A key step in the proof, which is of independent interest, is to show that any solution to the associated martingale problem is also a pathwise entropy solution to the stochastic PDE, a notion introduced in a recent series of papers.

# Generative compressed sensing with Fourier measurements

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Recently, it has been established that Deep Generative Models (DGMs) can be used as priors in inverse problems such as denoising, inpainting, medical and seismic imaging, and more. One inverse problem of tremendous interest since 2005 is compressed sensing (CS) – acquisition and provable recovery of sparse signals (or signals with low complexity) from a few, non-adaptive measurements. Recently DGMs have been proposed to replace the sparse signal model in CS, leading to theoretical guarantees and practical performance that improves on "classical compressed sensing" for classes of signals that can modelled well using DGMs when the measurement matrix and/or network weights follow a subgaussian distribution. We move beyond

the subgaussian assumption, to measurement matrices that are derived by sampling uniformly at random rows of a unitary matrix (including subsampled Fourier measurements as a special case). Specifically, we prove the first known restricted isometry guarantee for generative compressed sensing (GCS) with subsampled isometries, and provide recovery bounds with nearly order-optimal sample complexity. Furthermore, we extend our theory by constructing a model-adapted sampling strategy with an improved sample complexity.

# A smoothing technique for non-smooth optimization with applications to constrained minimax problems

KE YIN

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The minimax problems are an important class of optimization problems arising in image processing and machine learning. Efficient algorithms for solving strongly-convex-concave type of minimax problems have been proposed. However, more general cases, in particular the constrained minimax problems are much less studied in the literature. We propose a new framework for solving constrained semi-infinite minimax problems, based on a smoothing technique for the maximum function. The convergence property and the convergence rate are presented.

# Boundary integral equation solvers for layered-medium scattering problems

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This talk will introduce our recent works on developing fast and high-order boundary integral equation solvers for solving the wave scattering problems in a layered-medium. Both the windowed Green's function method and the perfectly-matched-layer based integral equation method will be discussed as well as the regularization for singular integral operators and the numerical discretization relying on a Chebyshev-based rectangular-polar solver. Numerical examples will be presented to show the accuracy and efficiency of the methods.

#### Operator learning for multiscale PDEs

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This talk concerns operator learning for multiscale partial differential equations, where preserving high-frequency information is critical. We develop neural operators based on hierarchical attention or dilated convolution that achieve state-of-the-art performance on multiscale operator learning tasks. Those neural operators enable efficient forward and inverse solutions for multiscale problems. We conduct experiments to evaluate the performance on various datasets, including the multiscale elliptic equation, its inverse problem, Navier-Stokes equation, and Helmholtz equation.

#### A novel stochastic interacting particle-field algorithm for solving the 3D parabolic-parabolic Keller-Segel chemotaxis system

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We propose an efficient stochastic interacting particle-field (SIPF) algorithm for computing aggregation patterns and near singular solutions of the parabolic-parabolic Keller-Segel (KS) chemotaxis system in 3D. Our approach approximates KS solutions using empirical measures of particles coupled with a smoother field variable computed through the spectral method. Numerical experiments confirm the convergence and self-adaptive nature of the SIPF algorithm to high gradient regions, which provides a low-cost approach to studying the emergence of finite time blowup in 3D using a small number of Fourier modes and varying initial mass. Furthermore, we present the recent development in applying the SIPF algorithm to simulate the cancer cell invasion process.

#### H(div)-conforming HDG methods for the stress-velocity formulation of the Stokes equations and the Navier-Stokes equations

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In this talk we present a pressure-robust and superconvergent HDG method in stress-velocity formulation for the Stokes equations and the Navier-Stokes equations with strongly symmetric stress. The stress and velocity are approximated using piecewise polynomial space of order k and H(div)-conforming space of order k+1, respectively, where k is the polynomial order. In contrast, the tangential trace of the velocity is approximated using piecewise polynomials of order k. The discrete  $H^1$ -stability is established for the discrete solution. The proposed formulation yields divergence-free velocity, but causes difficulties for the derivation of the pressure-independent error estimate given that the pressure variable is not employed explicitly in the discrete formulation. This difficulty can be overcome by observing that the  $L^2$ projection to the stress space has a nice commuting property. Moreover, superconvergence for velocity in discrete  $H^1$ -norm is obtained, with regard to the degrees of freedom of the globally coupled unknowns. Then the convergence of the discrete solution to the weak solution for the Navier-Stokes equations via the compactness argument is rigorously analyzed under minimal regularity assumption. The strong convergence for velocity and stress is proved. Importantly, the strong convergence for velocity in discrete  $H^1$ norm is achieved. Several numerical experiments are carried out to confirm the proposed theories.

#### Microscale hydrodynamic cloaking and shielding for electroosmotic model

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In this talk, we discuss the recent progress on microscale hydrodynamic cloaking and shielding for electro-osmotic model, which contain single-field cloaking and shielding, enhanced near-cloaking and multi-field cloaking. Based on

asymptotic analysis, layer potential techniques and optimal control theory, the core-shell structure contains annulus, confocal ellipses, slightly deformed annulus or confocal ellipses and more general geometry are considered. Especially, the conditions that can ensure the occurrence of above microscale hydrodynamic cloaking and shielding are established. Our theoretical findings are validated and supplemented by a variety of numerical results. (This talk is based on joint works with Zhiqiang Miao (HNU), and Hongyu Liu (CityU))

#### Perfectly matched layer method for the wave scattering problem by a step-like surface

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This talk is concerned with the convergence theory of perfectly matched layer (PML) method for wave scattering problems in a half plane bounded by a step-like surface. When a plane wave impinges upon the surface, the scattered waves compose of an outgoing radiative field and two known parts. The first part consists of two parallel reflected plane waves of different phases, which propagate in two different subregions separated by a half-line parallel to the wave direction. The second part stands for an outgoing corner-scattering field which is discontinuous and represented by a double-layer potential. A piecewise circular PML is defined by introducing two types of complex coordinates transformations in the two subregions, respectively. A PML variational problem is proposed to approximate the scattered waves. The exponential convergence of the PML solution is established by two results based on the technique of Cagniard-de Hoop (CDH) transform. First, we show that the discontinuous corner-scattering field decays exponentially in the PML. Second, we show that the transparent boundary condition (TBC) defined by the PML is an exponentially small perturbation of the original TBC defined by the radiation condition. Numerical examples validate the theory and demonstrate the effectiveness of the proposed PML.

#### Conductivity imaging using deep neural networks

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Conductivity imaging is a fundamental task in medical imaging and serves as a valuable tool for medical diagnosis. It is typically formulated as the problem of recovering the conductivity coefficient within second-order elliptic PDEs from various observational data. In this talk, we study the numerical analysis for identifying conductivity coefficient from internal observation. Generally, a regularized formulation consisting of a data fidelity term and a regularization is employed. This formulation is then discretized using the finite difference method, finite element methods, or deep neural networks. A key issue in this context is establishing a priori error estimates for the recovered conductivity distribution. We will discuss our recent findings on employing deep neural networks for this class of problems, effectively leveraging relevant stability estimates.

# Topologically protected waves in superhoneycomb structures

Yi Zhu

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In the past decade, a revolution in materials sciences has been brought about by the recognition of topology in characterizing the physical properties. These materials which support novel topologically protected waves are termed as topological materials. In this talk, I shall first review recent advances on mathematical aspects on waves propagating in these materials. And then I will focus on our recent works of waves in a specific medium with superhoneycomb structures. More specifically, we prove that the bulk structure admits double Dicac cones in its spectrum, and two branches of topological edge states are bifurcated under perturbations even with a PT symmetry.

#### 3 Contributed Talks

#### Multiscale model reduction for a nonlinear strain-limiting Cosserat elasticity model using GMsFEM

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We consider a nonlinear strain-limiting Cosserat elasticity model in heterogeneous media. This elasticity model contains additional rotational degrees of freedom and has a strain-limiting property, ensuring bounded linearized strains even under high stresses. Potential applications include solids and biological fibers. However, numerical simulation of the strain-limiting Cosserat elastic media is challenging due to the complexity of the model and the heterogeneities that often arise in applied problems. We propose multiscale model reduction approaches based on the Generalized Multiscale Finite Element Method (GMsFEM) to reduce the computational cost. We consider both offline and residual-driven online multiscale approaches. To handle the nonlinearity of the model, we apply Picard iterations. We solve twodimensional model problems in various heterogeneous media (perforated, composite, stochastically heterogeneous) with small and big strain-limiting parameters to test the proposed multiscale approaches. The results demonstrate that the multiscale approaches can significantly reduce the computational cost while maintaining high accuracy. Moreover, the online approach provides more accurate solutions than the offline one, while the online adaptive strategy requires fewer degrees of freedom than the uniform one to obtain similar accuracy.

Superpixel-based and spatially-regularized diffusion learning for unsupervised hyperspectral image clustering Kangning Cui\*, Ruoning Li, Sam Polk, Yinyi Lin, Hongsheng

ZHANG, JAMES MURPHY, ROBERT PLEMMONS, RAYMOND CHAN Department of Mathematics, City University of Hong Kong, Hong Kong *Email:* kangnicui2-c@my.cityu.edu.hk

Hyperspectral images (HSIs) provide exceptional spatial and spectral resolution of a scene, crucial for various remote sensing applications. However, the high dimensionality, presence of noise and outliers, and the need for precise labels of HSIs present significant challenges to the analysis of HSIs, motivating the development of performant HSI clustering algorithms.

This work introduces a novel unsupervised HSI clustering algorithm—Superpixel-based and Spatially-regularized Diffusion Learning (S<sup>2</sup>DL)—which addresses these challenges by incorporating rich spatial information encoded in HSIs into diffusion geometry-based clustering. S<sup>2</sup>DL employs the Entropy Rate Superpixel (ERS) segmentation technique to partition an image into superpixels, then constructs a spatially-regularized diffusion graph using the most representative high-density pixels. This approach reduces computational burden while preserving accuracy. Cluster modes, serving as exemplars for underlying cluster structure, are identified as the highest-density pixels farthest in diffusion distance from other highest-density pixels. These modes guide the labeling of the remaining representative pixels from ERS superpixels. Finally, majority voting is applied to the labels assigned within each superpixel to propagate labels to the rest of the image.

This spatial-spectral approach simultaneously simplifies graph construction, reduces computational cost, and improves clustering performance. S<sup>2</sup>DL's performance is illustrated with extensive experiments on four publicly available, real-world HSIs: Indian Pines, Salinas, Salinas A, and WHU-Hi. Additionally, we apply S<sup>2</sup>DL to landscape-scale, unsupervised mangrove species mapping in the Mai Po Nature Reserve, Hong Kong, using a Gaofen-5 HSI. The success of S<sup>2</sup>DL in these diverse numerical experiments indicates its efficacy on a wide range of important unsupervised remote sensing analysis tasks.

# Determining sources in the bioluminescence tomography problem

Minghui Ding

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In this talk, we focus on bioluminescence tomography (BLT) problem, which involves reconstructing bioluminescent signals from external measurements of the Cauchy data. As one kind of optical imaging, the BLT has many merits such as high signal-to-noise ratio, non-destructivity and cost-effectiveness etc., and has potential applications such as cancer diagnosis, drug discovery and development as well as gene therapies and so on. In the literature, BLT is extensively studied based on diffusion approximation (DA) equation, where the distribution of peak sources is to be reconstructed and no uniqueness of solutions is guaranteed without adequate a priori information. Motivated by the uniqueness issue, several theoretical results are explored. The major contributions in this work that are new to the literature are two-fold: first, we show the theoretical uniqueness of the BLT problem where the light sources are in the shape of  $C^2$  domains or polyhedral- or corona-shaped; second, we support our results with plenty of problem-orientated numerical experiments.

# Spectral-Spatial Classification Methods for Hyperspectral Images

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Hyperspectral image (HSI) analysis has become an active topic in the field of remote sensing in recent years. HSIs often have hundreds of spectral bands of different wavelengths captured by aircraft or satellites that record land coverage. As one of the most major tasks, identifying detailed classes of pixels has drawn broad attentions and brought about a wide variety of methods due to the enhancement in spectral and spatial resolution. Among them, semi-supervised methods with a small number of training samples have gained popularity due to the large amount of time and cost required for manual labeling. Therefore, a spectral and spatial-based algorithm for HSI classification is presented in this work for its effectiveness in achieving satisfactory

results with limited labeled data. Specifically, spatial information of HSI data is utilized in the pre-processing stage and the post-processing stage of 3-stage method. The pre-processing stage not only improves the coherence of pixels within the same class, mitigates the effects of noise in HSI to improve the representation of training pixels, but also reduces the execution time through the application of Principal Component Analysis. Consequently, the classification results obtained by support vector machines are improved owing to the refined and denoised features. Additionally, the post-processing stage aims to ensure spatial connectivity in the classification map and helps prevent misclassification of isolated pixels in the image. Findings from the experiment show that 3-stage is capable of significantly decreasing the quantity of training pixels while enhancing the accuracy of classification. As a result, it is of great practical significance since expert annotations are often expensive and difficult to collect. Further to this, the shape adaptive reconstruction is introduced as a variant for the pre-processing stage whose flexible selection of reconstruction window makes it more suitable for those classes manifested as elongated shapes with sharp corners.

#### Analysis of the staggered DG method for the quasi-Newtonian Stokes flows

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In this talk, I will present a divergence-free staggered discontinuous Galerkin (DG) method for quasi-Newtonian Stokes flow problems. The flux and the tensor gradient of the velocity are introduced as additional unknowns and the pressure variable is eliminated from the system via the incompressibility condition. Thanks to the subtle construction of the finite element spaces used in our staggered DG method, no additional numerical flux or stabilization terms are needed. The well-posedness of this scheme is based on the abstract theory for the non-linear twofold saddle point problems. A prior error analysis for all the involved unknowns is also provided. In addition, the proposed scheme can be hybridizable and the global problem only involves the trace variables, rendering the method computationally attractive. Finally, several numerical experiments are carried out to illustrate the performance of our scheme.

# Approximation of functions from Korobov spaces by shallow RELU neural networks

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The work is the first innovative approach and result about the approximability of shallow neural networks on Korobov spaces. During the talk, a dimensional independent rate of approximating functions from the Korobov space by ReLU shallow neural networks will be established. Following the first main result, a generalization error will be emphasized. Finally, a discussion on a specific example will be given as a justification for the novelty and sufficiency of the main results.

#### On inverse problems in predator-prey models

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In this talk, we consider the inverse problem of determining the coefficients of interaction terms within some Lotka-Volterra models, with support from boundary observation of its non-negative solutions. In the physical background, the solutions to the predator-prey model stand for the population densities for predator and prey and are non-negative, which is a critical challenge in our inverse problem study. We mainly focus on the unique identifiability issue and tackle it with the high-order variation method, a relatively new technique introduced by the second author and his collaborators. This method can ensure the positivity of solutions and has broader applicability in other physical models with non-negativity requirements. Our study improves this method by choosing a more general solution  $(u_0, v_0)$  to expand around, achieving recovery for all interaction terms. By this means, we improve on the previous results and apply this to physical models to recover coefficients concerning compression, prey attack, crowding, carrying capacity, and many other interaction factors in the system. Finally, we apply our results to study three specific cases: the hydra-effects model, the Holling-Tanner model and the classic Lotka-Volterra model.

# The axisymmetric solution of 2-dimensional compressible Euler equations with discontinuous swirl

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In this talk, we studied the shock formation to the axisymmetric solution of Euler equations with discontinuous non-zero swirl. We proved the lower bound estimate of density with the influence of swirl. Through the estimate of density and relative sup-norm of the solution, we obtain the local existence and shock formation theorem for the classical solution. Furthermore, we studied how the swirl influences the system and proved that the swirl will not prevent shock formation. All the results work for the Cauchy problem with large initial data.

# The bifurcation of Dirac points in photonic/phononic structures

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The developments of topological insulators have provided a new avenue of creating interface modes (or edge modes) in photonic/phononic structures. Such created modes have a distinct property of being topologically protected and are stable with respect to perturbations in certain classes. In this talk, we will first review mathematical results on the existence of interface modes, and then report recent results on the creation of an in-gap interface mode that is bifurcated from a Dirac point in various two-dimensional photonic/phononic structures.

# Multicontinuum homogenization for coupled flow and transport equations

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In this work, we present the derivation of a multicontinuum model for the

coupled flow and transport equations by applying the multicontinuum homogenization. We perform the multicontinuum expansion to both flow and transport solutions and formulate coupled constraint cell problems to capture the multiscale property, where oversampled regions are utilized to avoid boundary effects. Assuming the smoothness of macroscopic variables, we will obtain a multicontinuum system composed of macroscopic elliptic equations and convection-diffusion-reaction equations with homogenized effective properties. Finally, We present numerical results for various coefficient fields and boundary conditions to validate our proposed algorithm.

# Fast asymptotic rates of convergence for neural networks under the hard margin condition

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We study the classical binary classification problem for hypothesis spaces of Deep Neural Networks (DNNs), under a favourable "hard margin condition" which guarantees that the two classes are well separated. It is known that for hypothesis spaces such as those induced by RKHS or local polynomials, this assumption leads to an exponential decay of the excess risk, while no rates faster than  $\mathcal{O}(n^{-1})$  have been shown to hold for Neural Networks under that same assumption. By considering the square surrogate loss with a weight decay penalty, we prove a general risk bound which leads to asymptotic rates of convergence for general families of Neural Networks hypothesis spaces. As one consequence of this result, we show that for hypothesis spaces of fully connected DNNs, the excess risk can asymptotically attain polynomial rates of convergence of order  $\mathcal{O}(n^{-\alpha})$  for arbitrarily large  $\alpha > 0$ , and even exponential rates of convergence under additional favourable assumptions.

#### Asymptotic and invariant-domain preserving schemes for scalar conservation equations with stiff source terms and multiple equilibrium points

ALEXANDRE ERN, JEAN-LUC GUERMOND, ZUODONG WANG\* SERENA team, Centre Inria de Paris & Ecole des ponts ParisTech, France *Email:* zuodong.wang@inria.fr

We propose a prediction-correction method to approximate scalar conservation equations with stiff source terms having multiple (at least two) stable equilibrium points. The scheme combines a (reaction-free) transport substep followed by a (transport-free) reaction substep. The transport substep is approximated using the forward Euler method with continuous finite elements and graph viscosity. The reaction substep is approximated using an exponential integrator. The crucial idea of the paper is to use a mesh-dependent cutoff of the reaction time-scale in the reaction substep. We establish a bound on the entropy residual motivating the design of the scheme. We show that the proposed scheme is invariant-domain preserving under the same CFL restriction on the time step as in the nonreactive case. Numerical results in one and two space dimensions using linear, convex, and nonconvex fluxes with smooth and nonsmooth initial data in various regimes show that the proposed scheme is asymptotic preserving.

# A phase tomography model in adaptive optics system YUTONG WU\*, ROLAND WAGNER, RONNY RAMLAU, RAYMOND CHAN Department of Mathematics, City University of Hong Kong, Hong Kong *Email:* yutwu3-c@my.cityu.edu.hk

In ground-based astronomy, images of objects captured by ground-based telescopes often suffer from atmospheric turbulence, leading to degraded image quality. To address this challenge, adaptive optics techniques are commonly employed to correct distortions in the wavefront. In this presentation, we introduce an algorithm designed for phase super-resolution using a sequence of measurements, the wavefront gradient obtained from wavefront sensors in adaptive optics systems. Our approach considers tomography and incorporates the Taylor frozen flow hypothesis, leveraging knowledge of wind velocities within the standard ESO 3-layer atmospheric model. Our research emphasizes understanding turbulence characteristics in the  $H^{11/6}$  space, based

on Kolmogorov's Theorem. We propose the H2L2 model, incorporating the  $H^2$  norm as a regularization term, and provide an analysis of approximating  $H^{11/6}$  using  $H^2$  in the context of ground-based astronomy. Numerical simulations and visualizations are carried out to demonstrate the effectiveness of our approach.

#### Multicontinuum homogenization in perforated domains

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In this talk, we present a general framework for multicontinuum homogenization in perforated domains. The simulations of problems in perforated domains are expensive and, in many applications, coarse-grid macroscopic models are developed. Many previous approaches include homogenization, multiscale finite element methods, and so on. We design multicontinuum homogenization based on our recently proposed framework. In this setting, we distinguish different spatial regions in perforations based on their sizes. For example, very thin perforations are considered as one continua, while larger perforations are considered as another continua. Besides, we present a framework by formulating cell problems for each continuum using appropriate constraints for the solution averages and their gradients. These cell problem solutions are used in a multiscale expansion and in deriving novel macroscopic systems for multicontinuum homogenization. Our proposed approaches are designed for problems without scale separation. We present numerical results for two continuum problems and demonstrate the accuracy of the proposed methods.

#### Computing resonant modes for biperiodic structures

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Many photonic devices, such as photonic crystal slabs, cross gratings, and periodic metasurfaces, are biperiodic structures with two independent periodic directions, and are sandwiched between two homogeneous media. Many

applications of these devices are closely related to resonance phenomena. Therefore, efficient computation of resonant modes is crucial in device design and structure analysis. Since resonant modes satisfy outgoing radiation conditions, perfectly matched layers (PMLs) are usually used to truncate the unbounded spatial variable perpendicular to the periodic directions. In this paper, we develop an efficient method without using PMLs to calculate resonant modes in biperiodic structures. By a fast and memory-efficient direct solver, we reduce the original eigenvalue problem to a small matrix nonlinear eigenvalue problem which is solved by the contour integral method. Numerical examples show that our method is efficient with respect to memory usage and CPU time, free of spurious solutions, and determines degenerate resonant modes without any difficulty.

# Data-adaptive graph framelets with generalized vanishing moments for graph signal processing

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In this work, we propose a novel and general framework to construct tight framelet systems on graphs with localized supports based on hierarchical partitions. Our construction provides parametrized graph framelet systems with great generality based on partition trees, by which we are able to find the size of a low-dimensional subspace that best fits the low-rank structure of a family of signals. The orthogonal decomposition of subspaces provides a key ingredient for the definition of "generalized vanishing moments" for graph framelets. In a data-adaptive setting, the graph framelet systems can be learned by solving an optimization problem on Stiefel manifolds with respect to our parameterization. Moreover, such graph framelet systems can be further improved by solving a subsequent optimization problem on Stiefel manifolds, aiming at providing the utmost sparsity for a given family of graph signals. Experimental results show that our learned graph framelet systems perform superiorly in non-linear approximation and denoising tasks.

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