A fast and adaptive varying-scale morphological analysis method for rolling element bearing fault diagnosis
Changqing Shen, Qingbo He, Fanrang Kong and Peter W. Tse
DOI: 10.1177/0954406212460628
The online version of this article can be found at:
http://pic.sagepub.com/content/early/2012/09/23/0954406212460628

Published by:
SAGE
http://www.sagepublications.com

On behalf of:
Institution of Mechanical Engineers

Additional services and information for Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science can be found at:

Email Alerts: http://pic.sagepub.com/cgi/alerts
Subscriptions: http://pic.sagepub.com/subscriptions
Reprints: http://www.sagepub.com/journalsReprints.nav
Permissions: http://www.sagepub.com/journalsPermissions.nav

>> OnlineFirst Version of Record - Sep 25, 2012
What is This?
A fast and adaptive varying-scale morphological analysis method for rolling element bearing fault diagnosis

Changqing Shen¹,², Qingbo He², Fanrang Kong² and Peter W Tse¹

Abstract
The research in fault diagnosis for rolling element bearings has been attracting great interest in recent years. This is because bearings are frequently failed and the consequence could cause unexpected breakdown of machines. When a fault is occurring in a bearing, periodic impulses can be revealed in its generated vibration frequency spectrum. Different types of bearing faults will lead to impulses appearing at different periodic intervals. In order to extract the periodic impulses effectively, numerous techniques have been developed to reveal bearing fault characteristic frequencies. In this study, an adaptive varying-scale morphological analysis in time domain is proposed. This analysis can be applied to one-dimensional signal by defining different lengths of the structure elements based on the local peaks of the impulses. The analysis has been first validated by simulated impulses, and then by real bearing vibration signals embedded with faulty impulses caused by an inner race defect and an outer race defect. The results indicate that by using the proposed adaptive varying-scale morphological analysis, the cause of bearing defect could be accurately identified even the faulty impulses were partially covered by noise. Moreover, compared to other existing methods, the analysis can be functioned as an efficient faulty features extractor and performed in a very fast manner.

Keywords
Signal processing, bearing fault diagnosis, morphology, varying-scale feature extractor, vibration analysis

Date received: 20 May 2012; accepted: 14 August 2012

Introduction
The rolling element bearing is one of the most important components in rotating machines that are employed by aircraft engines, automobile transmission systems, power plants, and others. Faults occurred in bearings ultimately could cause unexpected breakdown of machines. Hence, the consequence could be significant loss in economy and in worst case, lead to human casualties. To avoid the occurrence of catastrophe and minimize the downtime of a defective machine, the fault diagnosis of bearings is of great meaning in industry and the research on the bearing fault diagnosis has drawn lots of attention during the past decades.¹,²

A bearing usually consists of an inner race, an outer race, a cage and a number of rollers. Once the surface of these components suffers local defect, there will be periodic impulses generated by the surface when it has contacted with another surface during the bearing is rotating. Consequently, periodic impulses are generated and recorded in the captured vibration signals. Hence, the impulses contain important information about the component health status and the extraction of faulty impulses is the most important task in vibration-based bearing fault diagnosis.³,⁴

In order to analyze the vibration signals, a variety of data-driven techniques were developed for bearing fault diagnosis. Time–frequency analysis, which can
reveal both information in time domain and frequency domain, was developed for non-stationary signals. Cohen\textsuperscript{7} reviewed several time–frequency distributions, such as Wigner–Ville distribution, Choi–Williams distribution, etc. Recently, a newer time–frequency analysis called wavelet transform (WT) has attracted a lot of attention in research. WT that can decompose the temporal raw signal into different scales with different frequency bandwidths has been widely applied in fault diagnosis.\textsuperscript{6} He et al.\textsuperscript{10} decomposed the mechanical watch signal and detected the singularity with wavelet decomposition. Rafiee et al.\textsuperscript{8} developed an automatic method for gear and bearing fault diagnosis by applying different mother wavelet functions to conduct the vibration signal decomposition. The ensemble empirical mode decomposition (EEMD) method has been recently proposed to decompose the signal into a set of intrinsic mode functions (IMFs) according to its own natural oscillatory mode.\textsuperscript{9} Lei et al.\textsuperscript{11} effectively applied the EEMD method to rub-impact fault diagnosis of a power generator and early rub-impact fault diagnosis of a heavy oil catalytic cracking machine set. Dong et al.\textsuperscript{12} employed an improved EMD method to decompose the raw bearing vibration signals to a number of IMFs and revealed the bearing faults by selecting the suitable IMF.

In recent years, a new method called morphological signal processing has drawn the attention of researchers in bearing fault diagnosis.\textsuperscript{13,14} Mathematical morphology is a kind of nonlinear analysis method that has been developed and applied into various areas successfully, such as image processing and signal analysis, etc. Nikolaou et al.\textsuperscript{15} analyzed the bearing vibration signal with a flat structure element (SE). Hao et al.\textsuperscript{16} applied the morphological un-decimated wavelet (MUDW) decomposition scheme to extract impulsive features generated by rolling element bearings that were overwhelmed by noise. Wang et al.\textsuperscript{17} proposed an improved morphological filter to extract the impulsive attenuation signals. Li et al.\textsuperscript{18} proposed a weighted multi-scale morphological gradient filter and a multi-scale fractal dimension based on morphological covering method\textsuperscript{19} for bearing and gear fault diagnosis, respectively. Li et al.\textsuperscript{20} proposed a multi-scale autocorrelation via morphological wavelet slices method to detect bearing fault signatures. Wang et al.\textsuperscript{21} presented a concept called ‘morphogram’ for determining the optimal length of flat structure element for enhancing the ability of bearing fault diagnosis. Although their works have certain degree of successful fault diagnosis, most of their proposed algorithms need some prior knowledge of the raw signal, such as the periodic interval of generated impulses or a fixed SE length in single-scale morphology analysis.

Multi-scale morphology analysis was employed by Zhang et al.\textsuperscript{22} to extract morphological features at different scales for bearing fault diagnosis. This analysis performed the morphological transforms with different lengths of SE that were determined by the signal and then calculated the mean value of the results. Multi-scale morphology analysis does not require any prior knowledge as listed above. However, it needs to repeat the morphological transform for many times with different scales. Hence, the processing of the analysis is time consuming. In summary, even several morphology based analyses had been developed; they have their own limitations in the application of bearing fault diagnosis.

In this study, an adaptive varying-scale morphology analysis method is introduced here. Different from the previous morphology analyses, the SEs for different signal temporal points are different from each other. The parameters of SEs are determined by the time interval between two adjacent local impulsive peaks. As a result, a number of SEs will be generated and each of them may have time length. That is, each SE is created to fit the local characteristics of impulsive peaks. A simplified frequency spectrum can then be generated so that the maintenance staff can easily identify the concerned bearing fault characteristic frequency, such as the outer race fault characteristic frequency, in the simplified spectrum. In summary, this proposed analysis aims to extract the bearing fault-related impulses so as to find the type of fault occurred in the inspected bearing. Compared to other morphology analyses, it needs little prior knowledge and does not require intensive calculations as that of the multi-scale morphology analysis.

The rest of this article is outlined as follows. The following section briefly describes the fundamental theory of mathematical morphology. Our proposed fast and adaptive varying-scale morphology theory is introduced in the later section, followed by the verification tests using simulated bearing impulsive signals as stated in ‘Simulation study’ section. The process in extracting bearing faulty features is also described here. The next section provides details in the validation of the performance of analysis using real roller bearings with an inner race and an outer race faults. The result of a comparison study is given to confirm the superior of this analysis in computational time. Finally, conclusions are drawn in the last section.

Mathematical theory of morphological analysis

Theoretical fundamental of morphological filter

The theory of the morphological filter with structure element was first introduced by Magaros and Schafer\textsuperscript{23} in 1987. Let \( f(n) \) be the one-dimensional discrete signal and \( g(n) \) be the structure element. The morphological filter is composed of four basic mathematical operators, namely the erosion, the
dilation, the opening and the closing operators as defined below:

Erosion:
\[ (f\ominus g)(n) = \min(f(n + m) - g(m)), \]
\[ m = 0, 1, 2 \ldots M - 1, n = 0, 1, 2 \ldots N - 1 \]  
\[ (1) \]

Dilation:
\[ (f\oplus g)(n) = \max(f(n - m) + g(m)), \]
\[ m = 0, 1, 2 \ldots M - 1, n = 0, 1, 2 \ldots N - 1 \]  
\[ (2) \]

Opening:
\[ (f \circ g)(n) = (f\ominus g \oplus g)(n), \]
\[ n = 0, 1, 2 \ldots N - 1 \]  
\[ (3) \]

Closing:
\[ (f \bullet g)(n) = (f \oplus g \ominus g)(n), \]
\[ n = 0, 1, 2 \ldots N - 1 \]  
\[ (4) \]

where \( \Theta, \oplus, \circ \) and \( \bullet \) denote the operators for erosion, dilation, opening and closing respectively. Examples that show the functions of the four operators in processing a time series are shown in Figure 1 and the flat structure element is adopted.

According to the filtering results of the examples, it is concluded that the erosion operator can smooth negative impulses and suppress positive impulses, whilst the dilation operator can smooth positive impulses and suppress negative impulses. The opening operator preserves negative impulses but levels positive impulses, whilst the closing operator preserves positive impulses but fills up the valleys.

The structure elements

Besides the functions of morphological operators, the SE is another key component of the morphological filters. There are three key factors of SE must be considered, namely the shape, height and length of SE. Generally, the shape of SE varies from regular to irregular curves, such as flat, triangle, semicircle, etc. Zhang et al.\textsuperscript{22} illustrated that the shapes of SE had little effect on analyzing temporal signal. Hence, the flat SE with zero amplitude or height is chosen for this study. The length of the SE is a vital factor in analyzing one dimensional temporal signal.

Figure 2 shows the results of morphological filtering when improper SE lengths were used. As shown in the left four diagrams of Figure 2, when too short a length of SE is used, there is nearly no change to the original temporal signal after applying the morphological filtering. On the other hand, when too long a length of SE is used, the impulses as shown in the right four diagrams of Figure 2 have been leveled. From this experiment, it has indicated the importance in selecting a proper length of SE and its length must be adaptive to the property of the inspected raw temporal signal. That is, in order to suppress noise and extract only the features of bearing faulty impulses, a special scheme must be used to select the most suitable length of SE for each type of periodic impulses.

Figure 1. Examples: (a) the original time series, time series processed by (b) the erosion operator, (c) the dilation operator, (d) the opening operator, and (e) the closing operator (the solid line is the original time series).

Figure 2. The results of using improper SEs lengths: (a)–(d) the filtered signals after using too short a length of SE for erosion, dilation, opening, and closing operators, respectively; (e)–(h) the filtered signals after using too long a length of SE for erosion, dilation, opening, and closing operators, respectively.
Principle of the varying-scale morphological analysis

Different from the idea of multi-scale morphological analysis which needs numerous iterations for morphology transforms and becomes computational intensive, this article presents an adaptive varying-scale morphological analysis that requires only one iteration for morphological transform.

It is pointed that the amplitudes of the noise are randomly distributed. Accordingly, the fault related impacts in the vibration signals are whelmed in various degrees. As a result, the effect brought by the noise varies with the time. Hence, to suppress the noise and extract the fault impulses effectively, the fluctuated effect brought by the noise should be considered during processing the vibration signals. To tackle this problem, the SEs should be dynamically determined and also varies with the time based on the above analysis. Let \( f(n) \) and \( g_n(k) \) denote a discrete one-dimensional signal and the required SE of morphological analysis, respectively. For a signal with a temporal length of \( N \), there will be \( N \) number of SEs denoted as \( g_0(k), g_1(k), \ldots, g_N(k) \). Hence, at different temporal points of the raw signal, the length of SEs that are employed to conduct the morphology transform will be varied from \( g_0(k) \) to \( g_N(k) \). The varying-scale morphology transform for the four operators can be re-defined by replacing \( g \) in equations (1) to (4) by \( g_n \) as shown in the followings.

Erosion:

\[
(f \ominus g_n)(n) = \min\{f(n + m) - g_n(m)\}, \quad m = 0, 1, 2 \ldots M - 1, \quad n = 0, 1, 2 \ldots N - 1
\]

Dilation:

\[
(f \oplus g_n)(n) = \max\{f(n - m) + g_n(m)\}, \quad m = 0, 1, 2 \ldots M - 1, \quad n = 0, 1, 2 \ldots N - 1
\]

Opening:

\[
(f \circ g_n)(n) = (f \ominus g_n \oplus g_n)(n), \quad n = 0, 1, 2 \ldots N - 1
\]

Closing:

\[
(f \bullet g_n)(n) = (f \oplus g_n \ominus g_n)(n), \quad n = 0, 1, 2 \ldots N - 1
\]

Based on the results shown in Figure 1, the four basic operators are capable of extracting the different characteristics of the same signal. It is seen that the closing operator extracts the positive impulses but fills up the valleys, while the opening operator preserves the negative impulse and levels the positive impacts. As a combination of the closing operator and the opening operator, the difference operator is able to extract both positive and negative fault impulses simultaneously and thus preserve more useful signatures. Hence it was adopted in this study. The difference operator is defined as follows:

\[
(f \bullet g_n - f \circ g_n)(n) = (f \oplus g_n \Theta g_n - f \Theta g_n \oplus g_n)(n)
\]  

(9)

The difference operator is expected to have better ability in identifying the faulty impulses by preserving and further enhancing them on the resultant waveform.

When a defective bearing is in rotating motion, its defected surfaces will interact with other surfaces such that faulty impulses will be generated periodically. The periodic impulses will be appeared in the temporal waveform of the captured vibration raw signal with local peaks as shown in Figure 3. Due to the existence of noise impulses, more local peaks will be formed in the vibration signals. Hence, the local peaks contain both fault related impulses and the information of noise distribution. Besides, the noises distribution varies at different time point. Consequently, the SE should be selected dynamically according to the noise distribution. Here, the temporal lengths between two adjacent local peaks are used to determine one of the varying lengths of SEs. The selected local length of SE is equal to the time interval between the two adjacent local peaks by using our proposed varying-scale morphological analysis.

Let \( V = \{V_i | i = 1, 2, 3, \ldots, N_p\} \) denote the local peaks which are the local maximum of the positive impulses in the temporal raw signal and their positions are indicated by \( P = \{P_i | i = 1, 2, 3, \ldots, N_p\} \). The time intervals between each pair of two adjacent local peaks can be calculated by \( S = \{S_i | S_i = P_{i+1} - P_i, i = 1, 2, 3, \ldots, N_p - 1\} \). Based on the above definitions, for a discrete signal point between the positions \( P_i \) and \( P_{i+1} \), the pair of two local adjacent peaks are \( V_i \) and \( V_{i+1} \), respectively. The time interval between them is the time distance \( S_i \).

\[ S_i = P_{i+1} - P_i \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]

\[ P_{i+1} \]

\[ V_i \]

\[ V_{i+1} \]

\[ S_i \]

\[ P_i \]
Let $\text{Peak}(n)$ denote the local peak nearest to the $n$th discrete signal in the set $P$.

$$\text{Peak}(n) = \max \{ P_{n} \mid P_{n} \in P \text{ and } P_{n} \leq n \},$$

$n = 0, 1, 2 \ldots N - 1$ \hspace{1cm} (10)

$I(n)$ is the index of $\text{Peak}(n)$ in set $P$. By considering all of the local peaks appear in the raw signal, the selected lengths of the SEs are defined as follows

$$\text{length}(g_{n}) = S_{h_{0}j}, \quad n = 0, 1, 2 \ldots N - 1$$ \hspace{1cm} (11)

Finally, the discrete one-dimensional raw signal has been morphological filtered by the difference operator and the adaptive varying-scale SEs. Based on the selected SEs, a new frequency spectrum will be generated. The bearing fault characteristic frequencies can be easily revealed from this new frequency spectrum. A flow chart that describes the step-by-step process of the varying-scale morphological analysis is shown in Figure 4.

**Simulation study**

To validate the performance of the varying-scale morphological analysis a simulated signal with 2048 samples and embedded with noise was used as follows

$$y(k) = x_{1}(k) + x_{2}(k), \quad k = 0, 1, 2 \ldots , 2047$$ \hspace{1cm} (12)

where $x_{2}(k)$ denotes the noise subjected to a normal Gaussian distribution with mean 0 and standard deviation 0.2, and $x_{1}(k)$ is the simulated vibration signal defined as

$$x_{1}(k) = e^{-900 \text{mod}(k/12000, 0.01)} \sin(17\pi k/60),$$

$k = 0, 1, 2, \ldots, 2047$ \hspace{1cm} (13)

As shown in Figure 5(a), the characteristic frequency of the simulated vibration signal with faulty impulses at 100 Hz. The sampling frequency of the signal was 12,000 Hz and 2048 samplings were used in this study. A randomly distributed noise was imposed to the simulated signal. The polluted signal and its spectrum are shown in Figure 5(b) and (c), respectively. The characteristic frequency and its harmonics cannot be clearly revealed in the spectrum.

The varying-scale morphological analysis was applied to the simulated signal. The results are shown in Figure 6. The lengths of SEs at different time intervals ranged from 2 to 9 sampling intervals for the random distribution of noise impulses. They were selected depended on the distribution of local peaks and the processed signal was shown as plotted in Figure 6(b). From this processed signal the fluctuation caused by noise was suppressed effectively. Hence, the bearing fault characteristic frequency (at 100 Hz) and its harmonics (at 200 Hz, 300 Hz, etc.) can be easily revealed in the new spectrum as shown in Figure 6(c).

Figure 7 shows the results produced by the multi-scale morphological analysis.20 Its filtered signal and its spectrum generated by filtered signal are shown in Figure 7(a) and (b), respectively. In comparison (Figures 6(c) and 7(b)), both methods obtained roughly similar results but the harmonics, especially the second harmonic, is less obvious than that generated by the varying-scale morphological analysis. Besides, the computation times of using the varying scale and multi scale morphological analyses are shown in Figure 8. Our analysis is more computationally efficient as it only took less than one-third of the time used by the multi-scale analysis.
Experimental validation

To further evaluate the varying-scale morphological analysis, the data set of bearing faults prepared by the Case Western Reserve University (CWRU) was used for verification purpose. The bearing data from CWRU was obtained from a rotating machine test rig and captured at a sampling frequency of 12 kHz and at the driving end bearing housing. The part number of the tested bearing is 6205-2RS JEM SKF and its specification is given in Table 1.

For rolling element bearings, let \( Z \) and \( d \) denote the number and the diameter of the rolling elements respectively, \( D_m \) and \( \alpha \) denote the pitch diameter and the contact angle of the rolling element bearing respectively, and \( f_n \) denotes the rotational frequency of the rotor in Hz and can be calculated by \( f_n = \frac{f_r}{60} \), where \( f_r \) is the rotating speed of rotor in rpm. The ball pass frequency of the outer race (BPFO) can be obtained by

\[
BPFO = \frac{1}{2} \left( 1 - \frac{d}{D_m} \cos \alpha \right) f_n Z
\]

If the surface of the outer race has a crack, then each time a rolling element passing through the crack will create periodic impulses with a time interval, \( \Delta t \) as

\[
\Delta t = \frac{1}{BPFO}
\]

Hence, an increase in magnitude of the outer race fault characteristic frequency, which is equivalent to BPFO, will reflect a defect occurred in the outer race of the bearing.

Similarly, the ball pass frequency in the inner race (BPFI) is

\[
BPFI = \frac{1}{2} \left( 1 + \frac{d}{D_m} \cos \alpha \right) f_n Z
\]

Therefore, the inner race fault characteristic frequency is equivalent to BPFI. An increase in magnitude of the BPFO will reflect a defect occurred in the inner race. The calculated bearing inner race and outer race fault characteristic frequencies are listed in Table 2.

The raw outer race fault vibration signal and its spectrum are displayed in Figure 9(a) and (b), respectively. After applying the varying-scale morphological analysis, Figure 10(a) and (b) show the lengths of SEs at different discrete signal points and the filtered signal, respectively. Figure 10(c) shows the frequency spectrum of the filtered signal. Note that after applying the adaptive filtering, the noise has been suppressed and the outer race fault characteristic frequency and its harmonics can be easily detected.

Comparing these results with that generated by the
multi-scale analysis as shown in Figure 11(a) and (b), the harmonics are not that obvious as those obtained by the varying-scale morphological analysis (Figure 10(c)). Moreover, it took less time for computing the results which is only one-third of that used by the multi-scale analysis as stated in Figure 8.

Figure 12(a) and (b) shows the raw inner race fault vibration signal and its spectrum, respectively.
morphological analysis.

The varying-scale SEs’ lengths, filtered signal and its spectrum are displayed in Figure 13(a), (b) and (c), respectively. The results indicate that the inner race fault characteristic frequency and its second harmonic can be more easily revealed by varying-scale morphological analysis as shown in Figure 13(c) than that by multi-scale analysis as shown in Figure 14(b). This time, the varying-scale morphological analysis used 56% less time than that used by the multi-scale analysis.

Conclusion

This article presents an adaptive varying-scale morphological analysis that can generate better result in bearing fault diagnosis than that obtained by the multi-scale morphological analysis but with significant improvement in computational time. It saves half to two-third of the time used by the multi-scale morphological analysis. Hence, this analysis is more suitable for on-line bearing fault diagnosis or fast diagnostic process. Moreover, it can effectively suppress noise and extract the faulty impulses generated by various bearing defects. Since this analysis can adaptively determine suitable SEs based on different time intervals of the local peaks by using varying-scale approach, it can be used to detect multiple bearing faults, such as outer and inner race defects co-existed in a bearing. Such research will be continued in future. Nevertheless, in cases when the original bearing fault signal has been heavily covered by noise and the signal-to-noise (S/N) ratio becomes too low, then a band-pass filter is needed to preprocess the signal and enhance its S/N ratio prior to the application of varying-scale morphological analysis. An important note is that the analysis does not require any prior knowledge of the periodic interval of impulses. It can reveal all possible intervals using varying-scale and then confirm the type of bearing fault that has occurred in the final frequency spectrum. Hence, the varying-scale morphological analysis is adaptive and fast in bearing fault diagnosis.

Funding

The work described in this paper was fully supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. CityU 122011) and the Natural Science Foundation of China (Grant No. 51075379).

Acknowledgment

The authors would also like to thank Professor KA Loparo of Case Western Reserve University for his kind permission to use their bearing data. The anonymous referees are also appreciated for their constructive comments.

References


---

**Appendix**

**Notation**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>diameter of the rolling elements (mm)</td>
</tr>
<tr>
<td>f_n</td>
<td>bearing rotational frequency (Hz)</td>
</tr>
<tr>
<td>f_r</td>
<td>rotating speed of rotor (r/min)</td>
</tr>
<tr>
<td>k</td>
<td>sample number of simulated signal</td>
</tr>
<tr>
<td>m</td>
<td>sample number of the raw signal</td>
</tr>
<tr>
<td>n</td>
<td>sample number of the structure element</td>
</tr>
<tr>
<td>x_1</td>
<td>simulated vibration signal with fault impulses at 100 Hz (V)</td>
</tr>
<tr>
<td>x_2</td>
<td>normal Gaussian distribution with mean 0 and standard deviation 0.2 (V)</td>
</tr>
<tr>
<td>y</td>
<td>simulated signal with fault impulses whelmed by noise (V)</td>
</tr>
<tr>
<td>D_m</td>
<td>pitch diameter of the bearing (mm)</td>
</tr>
<tr>
<td>M</td>
<td>sample length of the raw signal</td>
</tr>
<tr>
<td>N</td>
<td>sample length of the structure element</td>
</tr>
<tr>
<td>P</td>
<td>position number of the local maxima</td>
</tr>
<tr>
<td>S</td>
<td>interval between each pair of two adjacent local peaks</td>
</tr>
<tr>
<td>V</td>
<td>local maxima of the raw signal (V)</td>
</tr>
<tr>
<td>Z</td>
<td>number of the rolling elements</td>
</tr>
<tr>
<td>a</td>
<td>contact angle</td>
</tr>
</tbody>
</table>